AN

ESSAY

ON THE

STRENGTH AND STRESS OF TIMBER,

ETC. ETC.

AN

ESSAY

ON THE

STRENGTH AND STRESS OF TIMBER,

FOUNDED UPON

EXPERIMENTS

PERFORMED AT THE

ROYAL MILITARY ACADEMY,

ON SPECIMENS SELECTED FROM THE ROYAL ARSENAL, AND
HIS MAJESTY'S DOCK-YARD, WOOLWICH:

PRECEDED BY

AN HISTORICAL REVIEW

OF FORMER

THEORIES AND EXPERIMENTS;

WITH

NUMEROUS TABLES AND PLATES.

ALSO

AN APPENDIX,

ON THE

STRENGTH OF IRON, AND OTHER MATERIALS.

By PETER BARLOW, F.R.S.
OF THE ROYAL MILITARY ACADEMY;
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OF THE SOCIETY OF CIVIL ENGINEERS, ETC. ETC.

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TO

THE RIGHT HONOURABLE

EARL MULGRAVE,

MASTER-GENERAL OF THE ORDNANCE,

ETC. ETC. ETC.

MY LORD,

It was with your permission that I first engaged in the Experiments which have laid the foundation of the following Essay; and with your approbation that I prepared them for the press. Permit me then, my Lord, in submitting my labours to the Public, to acknowledge the obligation I am under for your Lordship's support and encouragement; and to express the sentiments of respect with which

I have the honour to be,

MY LORD,

Your Lordship's

Most obedient and humble Servant,

PETER BARLOW.

Royal Military Academy,
February 6th, 1824.

PREFACE.

A CORRECT knowledge of the Strength and Stress of Timber, and other materials, is admitted to be of the highest value to every one concerned in mechanical and architectural constructions: and yet it is generally allowed to be a part of those arts less understood than any other; and in which, therefore, the greatest errors are frequently committed.

"This subject," says Dr. Robison,* "is of so much importance, that in a nation so eminent for invention and ingenuity in every species of manufactory, and in particular so distinguished for its improvements in machinery of every kind, it is singular that no writer has treated of it in the detail which its difficulty and importance demand.

"The man of science who visits our great manufactories is delighted with the ingenuity which he observes in every part; the innumerable inventions which come even from the individual artisans, and the determined purpose of improvement and refinement which he sees in every workshop: every cotton-mill appears an academy of mechanical science; and mechanical invention is spreading from these fountains over the whole kingdom.

"But the philosopher is mortified to see this ardent spirit so cramped by ignorance of principles; and many of these

* Encyclopædia Britannica, Art. STRENGTH.

original and brilliant thoughts obscured and clogged with needless and even hurtful additions, and a complication of machinery which checks improvement even by its appearance of ingenuity: and there is nothing in which this want of scientific education is so frequently observed, as in the injudicious proportion of the parts; in which the strength and position are in nowise regulated by the strain to which they are exposed, and where repeated failures have been the only lessons."

It would be too much for me to presume that I have fully accomplished what this writer considers so great a desideratum; but I may, perhaps, be allowed to say, that I have made some considerable advances towards it, and put the practical engineer in possession of certain facts which have not before been generally known, and several rules for computation that he will in vain look for in any other work: for it is not only our country that has to complain of this paucity of information, there being no treatise, at least that I am acquainted with, in the French or any other language, from which such practical knowledge is to be obtained.

The French government has, it is true, at different times, ordered experiments to be carried on with a view to furnish the requisite data for a general theory: first, under the direction of Du Hamel and Buffon; and, latterly, under that of M. Girard, who afterwards published his "Traité Analytique de la Résistance des Solides:"* but I have shewn,

* A more recent course of Experiments, conducted with great accuracy and ability by M. le Chevalier Dupin, has been published in the tenth volume of the "Journal de l'Ecole Polytechnique," relating principally to the deflection of timber under various circumstances; and to which several references will be found in the following pages.

As I am apprehensive that in a paper lately presented to the *Académie Royale de l'Institute de France*, there is a slight error, arising out of a misunderstanding with reference to the origin of my experiments, it may be proper

I trust without prejudice, that he has failed of drawing from them those useful practical deductions we might have been led to expect.

The experiments of this author are numerous, and his investigations in many cases very ingenious: but by employing a calculus much too refined for the nature of the subject, and at the same time adopting the hypothesis of Mariotte and Leibnitz, he arrives at the same erroneous conclusions; making the strength of a beam, under certain circumstances, three times what it is in others, while experiment shews it to be stronger in the latter case than in the former.

A knowledge of all these inaccuracies and uncertainties, and a just appreciation of the value of correct information on a subject of so much importance, both in a civil and military point of view, particularly as it related to the education of the gentlemen designed for the corps of Royal Engineers and Artillery, induced General Mudge, lieutenant-governor of the Royal Military Academy, to propose my undertaking such a course of experiments as might, in combination with those above referred to, furnish a theory founded on the basis of practical data, wholly independent of any physical hypothesis.

My object, therefore, has been not only to furnish a new series of data, but also to make such experiments as might

to state, that they were begun in the summer of 1815; and that in 1816 I found myself much perplexed with the disagreement of my results and those of M. Girard; a circumstance which led me into the course of experiments detailed at p. 129 *et seq.* And it was not till after these were completed, that I was favoured with M. Dupin's memoir by Dr. Gregory, who had then recently received it from Paris; and in the perusal of which I felt great satisfaction by finding my results verified by those of the above gentleman.

lead to an explanation of the causes of those apparent anomalies which were observed in the results of preceding writers, and which rendered the whole of them doubtful and uncertain.

In this respect I have succeeded beyond my first expectation; having shewn that under a more rigid investigation many of these irregularities disappear; and that the several results, instead of contradicting each other, are in fact a confirmation of the accuracy with which they have been performed: and I should hope, that the same circumstance will be considered a strong indication of the truth of that theory which reconciles so many apparent incongruities.

But, in order to accomplish this, I have been under the necessity of rejecting several principles, or rather suppositions, hitherto generally made and admitted, and greatly to modify such as have been retained. To mention a few cases: I have shewn that the strain upon a beam fixed with one end in a wall and loaded at the other, is not simply as the length into the weight, but as the weight, length, and cosine of the angle of deflection; that is, as $l \times w \times cos. d$, where l denotes the length, w the weight, and d the angle of deflection; and that, when the same beam is supported at its two ends, and loaded in the middle, the strain is as $\frac{\frac{1}{2} l \times \frac{1}{2} W}{cos.^2 d'}$, or $\frac{l W}{4 \, cos.^2 d'}$; therefore, the strengths in the two cases, instead of being in the ratio of 1 to 4, are in the ratio of $1 : 4 \; cos. \; d \; cos. \; d'$; which, in many cases, approaches towards the ratio of 1 to 3.

Again, I have demonstrated that the strain being the same in these two cases, the element of deflection will be, in the former instance, double of that in the latter; and, as a necessary consequence of this, that the strength of a beam fixed

at each end is to that of the same when merely supported, in the ratio of 3 to 2; whereas former theories make the strengths in these two cases as 2 to 1.

These simple modifications reconcile all the apparent irregularities in the experiments of MM. Petit, Parent, and Belidor.

Again, as the strains upon beams supported at each end are as $\frac{l\,W}{cos.^2\,d}$; and as the ultimate deflection is as the square of the length, the strain increases faster than in the simple ratio of the length; or, which amounts to nearly the same, the strength decreases faster than in the inverse ratio of the length; which explains, either wholly or in part, the apparent anomalies in the valuable experiments of M. Buffon.

I have likewise shewn, both from experiment and theory, that beams fixed at any angle of inclination have the strain upon them diminished in the ratio of *rad.* : *cos.* I (I being the angle of inclination), or that their strength is increased in the ratio of *cos.* I to *radius;* whereas, it has commonly been said to increase in the ratio of $cos.^2$ I to $rad.^2$, and some writers have even made it as $cos.^3$ I to $rad.^3$ It also appears, that it is not the angle at which the beam is originally fixed, but that to which it is deflected, that must be adopted in our computation; and that, in many cases, a beam fixed at an angle of inclination upwards, will be broken with a less weight than an equal beam fixed horizontally; as is shewn in several of the experiments in Part II.

All these deductions have been obtained simply from a more accurate examination and valuation of the strains a beam is exposed to, according to the manner in which it is supported or fixed, and the direction of the exciting forces; and is, therefore, wholly independent of any par-

ticular theory of resistance; which is a part of this subject still more defective than that which relates to the strain: the latter only required slight modifications; but the former was, from beginning to end, erroneous.

The two principal theories of resistances are those of Galileo and Leibnitz, and these both contain one common error; viz. each of those authors considers bodies to be incompressible, and therefore, that every fibre in them, when strained, is in a state of tension; and which, according to the former, are all acting with equal energies; and, according to the latter, their reaction is proportional to the quantity of extension. The assumption of these different laws of tension necessarily leads to considerable difference in their ultimate results, as they depend upon the force of direct cohesion; the one making the strength of the same beam double that of the other.

It follows, also, as a necessary consequence of the assumption of incompressibility, that the former makes the strength of a triangular beam supported at each end with its edge upwards, double that of the same, or of an equal beam, with its base upwards; and according to the latter, the strength in the two cases will be as 3 to 1; whereas experiment shews it to be strongest in the latter position: and the same inconsistency runs through every comparison between the strength of differently formed beams, or of the same formed beam in different positions.

Being thus well aware of the defects of former theories, and of the danger of building a new one upon other physical assumptions, which, though more plausible, might be equally erroneous, I resolved to introduce no data but such as I obtained from actual experiment; and it is by this means that I have arrived at my general theorems, which may be applied with facility to every case; and so far as I have

been able to compare them with my own experiments, and with those of others of a similar kind, they give results as nearly correct as can possibly be expected, where so many irregularities constantly occur in cases which are, notwithstanding, to all external appearances precisely the same.

Having said thus much with respect to my own experiments, it remains for me to acknowledge my obligation for many others with which I have been favoured; particularly those of John Peake, esq. and M. Barrallier, carried on in His Majesty's Dock-yard, Woolwich; those of Colonel Beaufoy, performed in His Majesty's Dock-yard, Deptford; and a valuable course of various experiments, the result of many years' labour and observation, by Mr. Benjamin Couch, Master Mast-maker in His Majesty's Dock-yard, Plymouth. I am also greatly indebted to Thomas Telford, esq. for his highly interesting experiments on the strength of iron and iron wire; as I am also to Capt. Brown, the ingenious inventor of iron cables, for an account of his experiments of a similar kind.

I have already expressed my obligations to his Lordship, the Master-general of the Ordnance, for his encouragement and support; and I have a similar duty to perform towards the Honourable the principal Officers and Commissioners of His Majesty's Navy, for the permission which they gave me of selecting specimens proper for my purpose from the timber stores of Woolwich Dock-yard: in which selection I was much assisted by the extensive practical knowledge of many of the officers in that establishment, particularly Mr. Hookey, Assistant Builder, whose name frequently occurs in the following pages.

In conclusion, I beg to offer my sincere thanks to General Mudge, on whose suggestion the experiments were first

undertaken, for the assistance he has rendered me, not only in affording me every facility with respect to materials and workmanship, but also for the interest which he has himself taken in the progress of the experiments, and for the suggestions he very frequently offered with respect to the best modes of carrying them into execution. I am also, in the latter respect, much indebted to Dr. Gregory, who was present at the performance of many of the experiments, and who took great interest in their general progress.

Royal Military Academy,
October, 1817.

PREFACE

TO

THE THIRD EDITION.

THE favourable reception which the former editions of this work experienced from the public in general, and from gentlemen connected with the profession of Civil Engineers in particular, has induced me, in preparing it again for publication, to examine the whole with great attention, in order to correct such errors, either of my own or of the press, as had escaped my notice in the first impressions. These were not numerous; but all that were detected have been now carefully corrected. In the Appendix to the present edition, I have also introduced some new matter, which will, I hope, render the work still more serviceable; for this I am principally indebted to the kindness of some of my friends. M. J. BRUNEL, esq. has furnished me with some very accurate experiments made by him on the strength of Yorkshire iron and on cements; and, with the permission of J. RENNIE, esq., I have copied, almost entire, from the Philosophical Transactions, his valuable experiments on the transverse and direct strength of various materials, and their resistance to a crushing and to a wrenching strain. I have also added a few results obtained from a mean of several accurate experiments on the strength of ropes and cables, for which I am indebted to J. KNOWLES, esq. Secretary to the Sur-

veyors of the Navy. And to T. TREDGOLD, esq. I am highly indebted, for the several useful practical problems immediately preceding the Appendix, art. 149. With these additions and a careful revision of the whole work, I am in hopes this Essay may continue to merit the confidence and approbation of Architects and Engineers, for whose use it is principally designed.

ROYAL MILITARY ACADEMY,
January 16*th*, 1826.

CONTENTS

PART I.

Historical Sketch of Former Experiments on the direct Cohesion of different Woods.

	Art.
EMERSON's experiments on direct cohesion	3
Musschenbroeck's ditto	4
Buffon's and Du Hamel's observations with respect to the strength of different parts of the same tree, &c...........	5
Couch's experiments on the specific gravities of different woods at the butt and top	8
Couch's experiments on the seasoning of English oak	9
Hookey on the same subject	10
General remarks of Du Hamel with respect to the influence of soil on the strength and qualities of timber..............	11

Former Theories relative to the Transverse Strength and Stress of Beams.

Theory of Galileo	13
General results of this theory	16
Theories of Leibnitz and Mariotte	18
Galileo's and Leibnitz's theories compared	21
Bernoulli's remarks on the above theories	23
Theories of Euler, Legrange, and Dr. Robison	24

Experiments on the Transverse Strength.

Experiments by Parent and Belidor	28
Experiments of Du Hamel and Buffon	32
Experiments by Colonel Beaufoy..........................	34
Mean results of ditto..................................	35
Experiments by J. Peake, esq. and M. Barrallier	37
Table I. On Riga fir, kept dry.	
Table II. On Riga fir, kept wet.	
Table III. On various kinds of wood.	
Mean results, compared with those of Colonel Beaufoy	38

Art.

Experiments by Mr. Couch 39
Table I. On triangular oak beams fixed at one end.
Table II. Ditto, ditto, fixed at both ends.
Table III. On Riga, Norway, and Halifax spars, red and yellow pine, &c.

Of the principal Theories and Experiments on the Pressure and Longitudinal Resistance of Columns.

Euler's investigations 44
Lagrange's investigations 45
Girard's Traité Analytique de la Résistance des Solides 47
Table I. Experiments on the vertical pressure and resistance of columns 54
Table II. On the deflection of oak beams 55
Note, relative to the experiments of M. Dupin.

PART II.

Containing the Detail of Experiments, &c.

Description of apparatus, &c. for experiments on the direct cohesion 56
Table I. Experiments on the direct cohesion of different woods .... 58 *et seq.*
Table II. Experiments on the lateral adhesion of fir 61
Mean results of the above 62
The method of making the experiments on the transverse strength explained 64
Table III. Experiments on fir battens supported at each end .... 67 *et seq.*
Observations on the preceding experiments 73
Table IV. On battens cross-cut in the centre 74
Table V. On battens grooved from end to end 75
Table VI. On triangular fir battens 76
Table VII. On fir battens fixed at each end 77
Table VIII. On fir battens fixed at one end, at different angles of inclination 78
Table IX. On oak battens supported at each end 79
Table X. Ditto ditto 80
Table XI. On ash battens fixed at one end, at different angles of inclination 81
Table XII. Ditto on beech battens 82
Table XIII. On solid and hollow cylinders 83
Similar experiments to those of Table XII. on elm and teak .. 84

PART III.

THEORETICAL DEDUCTIONS.

Of the Deflection of Beams exposed to a Transverse Strain.

Art.

The deflection in terms of the length 86

On the elastic curve 87

The same, investigated on other principles 88

Of the deflection when the weight is uniformly distributed .. 90

The same law obtains with beams supported at each end 91

Of the deflection in terms of the breadth and depth 93

Comparison of the above deductions with M. Girard's experimental results 94

Experiments made with a view to this comparison 95

Other experiments of a similar kind 97

M. Dupin's experiments 101

Practical deductions 103

On the Transverse Strain of Beams.

The strain on a beam fixed at one end is as the length, weight, and cosine of deflection 104

The strain on a beam supported at each end, loaded in the middle, is as the length, weight, and square of the secant of deflection 105

The strain on the centre of a beam fixed at both ends is to the strain on the same, when only supported at the ends, as 2 : 3 107

Investigations of the strain when the beam is loaded with several weights, and in different parts of its length........ 108

Investigations relative to the strain of beams fixed at different angles of inclination 113

General theorems, deduced from a comparison of the strains and resistances 115

Comparison of the above theorems with former experimental results 116

Of the Position of the Neutral Axis, &c.

Investigations relative to the determination of the centre of tension 118

The centre of tension coincides with the centre of gravity 125

The above coincidence examined on physical principles...... 127

Investigations relative to the centre of compression 129

Art.

The centre of compression coincides with the centre of gravity of the area of compression 134
General theorem for the determination of the position of the neutral axis 135
The coincidence in Art. 134 examined on physical principles 137
Modification of the above general theorem 140
A series of experiments, on specimens selected from His Majesty's dock-yard, Woolwich, on various species of timber 142
Note.—Relative to the Mar-forest fir and larch.
General table of data 143
Experiments on bent timber........................ 145
Ditto on timber boiled and steamed........................ 147
Ditto on trussed and plain girders 148

PART IV.

Application of preceding Deductions to Practical Purposes.

On the forms proper to be given to beams, that they may be of equal resistance throughout 149
On the springs of carriages 151
Solution of practical problems 153
Comparison of experiments 154

APPENDIX.

Experiments on the resistance of iron wire 1
Ditto on the vertical strength of iron wire 3
Ditto on the rupture of the same by momentum............ 4
Ditto on iron bars and bolts 5
Ditto on Yorkshire iron, by M. J. Brunel 9
Ditto on the cohesive powers of different metals............ 10
Ditto on iron chains and cables 11
Ditto on the transverse strength of cast iron 15
Comparison of the strength of oak and iron 20
Experiments on the transverse strength of malleable iron 21
Ditto on the resistance of iron to a wrenching force 22
Ditto on different metals 26
Ditto on the resistance of cast iron to pressure 27
Ditto on different metals 28
Ditto on various materials........................ 29
Theoretical investigations connected with suspension bridges.. 30
On Revetments 33

AN ESSAY

ON THE

STRENGTH AND STRESS OF TIMBER.

PART I.

An Historical Sketch of Former Theories and Experiments.

1. THERE are four distinct strains to which a beam of timber, a bar of metal, and indeed every hard body, may be exposed, and in which the mechanical effort to produce the fracture, and the resistance opposed to it by the fibres or particles, are differently exerted; while each of these again is subject to various modifications, according to the manner in which the bodies are supported or fixed, the positions in which they are placed, and the direction of the forces or strains to which they are exposed.

These four distinct cases or strains may be stated as follow:

1. A body may be torn asunder by a stretching force applied in the direction of its fibres, as in the case of ropes, stretchers, king-posts, tye-beams, &c.

2. It may be broken across by a transverse strain, or by a force acting either perpendicularly or

obliquely to its length, as in the case of levers, joists, &c.

3. A beam or bar may also be crushed by a pressure exerted in the direction of its length, as in the case of pillars, posts, and truss-beams.

4. It may be twisted or wrenched by a force acting in a perpendicular direction, at the extremity of a lever or otherwise, as in the case of the axle of a wheel, the nail of a press, &c.

These several cases will form the subject of inquiry in the following pages.

Former Experiments on the direct Cohesion of Bodies, with various miscellaneous Observations and Remarks.

2. It is usual to distinguish by the expression *force of direct cohesion of bodies*, or simply *direct cohesion*, that force by which the fibres or particles of a body resist a separation, and which must ultimately be traced to that unknown cause we are accustomed to speak of under the denomination of *corpuscular attraction*.

This is by far the simplest strain of the four above alluded to with regard to its mechanical action; but the most difficult to submit to experiment, in consequence of the enormous forces that are requisite to produce the rupture even on pieces of small dimensions, and the great care that is necessary in applying those forces in the direct line of the fibres of the body; as, otherwise, the first rupture may be occasioned by some unequal action of the weight on a part of the fibres only, or by some force

of torsion whereby a part of them may be wrenched asunder.

The consequence in either case is, that the force of direct cohesion will be estimated at less than its real value; and it is probably owing to this circumstance that so little agreement is found in the results of such experiments as have been made with a view to this determination. These, however, are not very numerous; those of Musschenbroeck and Emerson being the principal ones that have been referred to as standards; although a few, on a limited scale, were made by MM. Petit and Parent, of the French Academy of Sciences.

3. Emerson does not specify the ultimate force of direct cohesion in the different woods which he has tabulated, but the load which may be safely borne by a square inch rod of each; so that we have not an opportunity of comparing his results with those of Musschenbroeck: nor does he describe the nature of his apparatus, nor the method of performing his experiments; and we are, therefore, unable to judge of their accuracy, and consequently, of the dependence which may be placed upon them in any practical application. It will be sufficient to observe, therefore, that they all fall very short of the ultimate strength of the materials to which they refer; but as it is intended to put the reader in possession of all that has been done relative to this subject, it will be proper to state his results; which are as follow:

	Pounds Avoirdupois.
Iron rod, an inch square, will bear......	76,400
Brass..............................	35,600
Hempen rope.......................	19,600

	Pounds Avoirdupois.
Ivory	15,700
Oak, box, yew, plum-tree	7,850
Elm, ash, beech	6,070
Walnut, plum	5,360
Red-fir, holly, elder, plane, crab	5,000
Cherry, hazel	4,760
Alder, asp, birch, willow	4,290
Lead	430
Free-stone	914

He gives also the following practical rule, viz. that a cylinder whose diameter is d inches, loaded to one-fourth of its absolute strength, will carry as follows:

	Cwt.
Iron	$135 \times d^2$
Good rope	$22 \times d^2$
Oak	$14 \times d^2$
Fir	$9 \times d^2$

He has also a few other practical deductions: but as we are inclined to believe, from a repetition of some of his experiments, that they were made in a gross, if not in a careless manner, it would be useless to give them in this place.

4. The experiments of Musschenbroeck were, on the contrary, made with considerable care; and he has been very minute in describing the apparatus which he employed, and the methods he adopted, so that much confidence has been placed in his results: his pieces, however, were very small; and a very trifling deviation from the dimensions which he supposed them to have, would make a great dif-

ference when converted into rods of an inch section; and he does not appear to have used any particular degree of caution in ascertaining their exact measure.

It is not, therefore, improbable that some of his rods exceeded the dimensions he ascribes to them. There seems at least no other way of accounting for the great strength which he attributes to some of his woods, particularly the oak, which is nearly double of what it was found in our experiments; as it is also double of what was found by MM. Petit and Parent, who state that 60 lbs. would just tear asunder a square line of sound oak. This gives 8640 lbs. for the utmost strength of a rod an inch square; which agrees very nearly, when reduced to English weight, with the experiments alluded to.

Musschenbroeck has given a very minute detail of the experiments on ash and walnut, stating the weights which were required to tear asunder slips taken from the four sides of the tree, and on each side in a regular succession from the centre to the circumference. His pieces were all formed into slips fitted to his apparatus, and cut down to the form of parallelopipedons of $\frac{1}{5}$th an inch square, and therefore $\frac{1}{25}$th of a square inch section; and the several weights required to produce the rupture when the rods are reduced to a square inch, are as stated in the following table:

	lbs.		*lbs.*
Locust-tree	20,100	Alder	13,900
Jugeb	18,500	Elm	13,200
Beech, oak	17,300	Mulberry	12,500
Orange	15,500	Willow	12,500

	lbs.		lbs.
Ash	12,000	Walnut	8,130
Plum	11,800	Pitch-pine	7,650
Elder	10,000	Quince	6,750
Pomegranate	9,750	Cypress	6,000
Lemon	9,250	Poplar	5,500
Tamarind	8,750	*Cedar	4,880
Fir	8,330		

The above are the principal experiments that have been made relative to the direct cohesion of timber, prior to those which are enumerated in a subsequent chapter: we shall, therefore, merely add to the preceding the remarks that have been made by Buffon, Musschenbroeck, &c. as to the strength of different parts of the same tree.

5. Buffon's experiments were made on a large scale, but not of the kind spoken of above; being principally directed to ascertaining the transverse strength: but as this, doubtless, depends ultimately upon the force of direct cohesion, his authority in this respect is very important. According to this philosopher, the heart of a tree is the heaviest and strongest, while Musschenbroeck found the reverse to be the case; viz. that the wood immediately surrounding the pith or heart was the weakest: and Dr. Robison asserts, under the article STRENGTH, "Encyclopædia Britannica," from his own observation on *very large* oaks and firs, that the heart was weaker than the exterior parts. He observes also,

* See Musschenbroeck's System of Natural Philosophy, published after his death by Lulofs, 3 vols. 4to; or, the French translation of the same, by Sigaud de la Fond, Paris, 1760.

that the wood next the bark, commonly called the *white*, or *sap*, is again weaker than the rest; and that, generally, the greatest strength is found between the centre and the sap.

6. With regard to our experiments, they were not particularly directed towards this inquiry; but, as far as observations were made, they rather agreed with the experiments of Buffon than with those of Musschenbroeck and Dr. Robison. In most cases the heaviest wood was found the strongest; and this was generally the case with those parts that grew nearest the centre of the trunk, and nearest to the root, provided it was so far removed from the latter as not to be very cross-grained. And M. Girard* is of the same opinion, stating it as a well-established fact, that the strongest part of a tree is nearest the centre.

7. From this contrariety of results, it is difficult to draw any satisfactory conclusion: the probability is, that much depends upon the age of the timber. While the tree is advancing in its growth, the last-formed wood, that is, the exterior parts, are probably weaker than the heart; but when a tree has attained complete maturity, and approaches, though imperceptibly, towards decay, the circumstances may be reversed; the exterior parts, or last-formed wood, becoming harder and stronger, while the central parts are beginning to experience that dissolution which ultimately pervades the whole. It will be observed that Dr. Robison states his timbers to

* Traité Analytique de la Résistance des Solides.

be *very large;* and Musschenbroeck's must have likewise been of considerable size, from the number of slips he was able to cut out between the centre and circumference: both which circumstances seem to give a degree of probability to the above suggestions.

Very nearly the same view is taken of this subject by Du Hamel, in his work, "Sur l'Exploitation des Bois," where the same ideas are given, not (as those above) merely as conjectures, but as facts, drawn from numerous experiments and observations. The author concludes his chapter on this subject as follows: "Si ce que nous venons "d'avancer est vrai, il faut nécessairement que le "bois qui est vers le centre du pied d'un arbre, "encore en crûe, soit plus pesant que celui qui est "au haut de la tige, et dans toutes les parties "de l'arbre; que celui qui est au centre, doit être "plus pesant que celui qui est à la circonférence. "Au contraire, quand les arbres sont sur leur "retour, le bois du centre doit être moins pesant "que celui qui est plus près de la superficie, à "cause de l'altération qu'il a soufferte. C'est un "fait que nous avons vérifié par plusieurs expé-"riences."

The work above referred to by Du Hamel contains many very curious and interesting experiments connected with this subject, as to the chemical analysis and natural decomposition of wood; of the quality of different woods, as depending upon the nature of the soil, &c.

From a great number of experiments and observations on the latter point, the author concludes that the best oaks, elms, and other great trees, are

the produce of *good lands*, rather of a dry than of a moist quality: they have a fine and clear bark; the sap is thinner in proportion to the diameter of the trunk; the ligneous layers are less thick, but are more adherent the one to the other, and have a greater uniformity of texture, than trees which grow in moister situations. The grain of these woods is fine and compact; and when they are examined with a good glass, their pores are observed to be filled with a species of varnish, or glutinous matter, strongly adherent, which gives them commonly a pale yellow colour, by which they may be distinguished from trees that are the growth of a different soil.

Also, in consequence of the closeness of their pores, they are more dense and heavy, become extremely hard, and resist the attack of worms.

The specific gravity of a tree growing in such soil as that above described, is to that of a similar tree in a wet marshy situation, frequently as 7 to 5; and the weights which a similar beam will support without breaking, in the two cases, are in about the ratio of 5 to 4.

May not this account for the superior quality of the Sussex oak? which I am informed by Mr. Hookey, timber-master in Deptford Dock-yard, he has always found to be the best for strength and durability: that the next in quality is that which grows in the south-west parts of Kent, and the north-east parts of Hampshire.

8. As to the density of the top and bottom of the same tree, and of the centre and external parts, much depends upon the age of the timber when

felled; but, generally, in a sound tree, the density is found to decrease from the butt upwards, and from the centre to the circumference. On the former point, the following experiments, the labour of many years, which have been made with great care by Mr. B. Couch, timber-master in His Majesty's Dock-yard, Plymouth, are highly valuable; and they are given in preference to those of Du Hamel; not only on account of their containing a greater variety of woods, but because the results are given in weights and measures which are more familiar to English engineers.

TABLE OF EXPERIMENTS,

Instituted in order to ascertain the Weight of a Cubic Foot of different Kinds of Wood; the Foreign when first imported, those of the Growth of England when felled: also, the Weight of each when fully seasoned; shewing, at the same time, the Loss sustained in Dimensions during the Process of Seasoning.

By Mr. Benjamin Couch, *of His Majesty's Dock-Yard, Plymouth.*

SPECIES (*in the language of commerce*).	*Country where produced.*	*What part of the Tree the pieces experimented on were cut from.*	DIMENSIONS.						Weight in air of a cubic foot, oz. avoirdupois.	
			When first planed for experiment.			*When seasoned.*				
			Length.		*Breadth and thickness, or diameter.*	*Length.*		*Breadth and thickness, or diameter.*	*When first planed for experiment.*	*When seasoned.*
			Ft.	*In.*	*Inches.*	*Ft.*	*In.*	*Inches.*	*Ounces.*	*Ounces.*
Riga Masts, superior	Russia	*Butt	7	6	18 diameter†	7	6	17¾ diameter	672	644
		Top	4	5	11 by 11	4	5	10 13/16 by 10¾	546	552‡
Riga Masts, inferior	Russia	Butt	12	0	12 diameter	12	0	11⅞ diameter	577	494
		Top	6	6	8¾ diameter	6	6	8½ diameter	464	464
Pitch Pine Mast	Baltimore	Butt	2	6	10 by 10	2	6	9¾ by 9¾	755	741
	North America	Top	6	0	7½ by 7½	6	0	7⅜ by 7⅜	518	524

* The butts and tops were cut from the same tree.

† When diameter is expressed, the pieces are cylindrical; all the others are parallelopipedons.

‡ Should it be asked, Why a cubic foot of some of the pieces increases in weight in *seasoning?* the reason is, that they lost more in dimensions than in weight in undergoing that *process.*

Columns 5, 6, 7, &c. will possibly be found of advantage to practical men, as they will enable them to form an idea of the decrease of dimensions in seasoning.

TABLE—(*continued.*)

SPECIES (*in the language of commerce*).	*Country where produced.*	*What part of the Tree the pieces experimented on were cut from.*	DIMENSIONS. *When first planed for experiment.* Length.		*When first planed for experiment.* Breadth and thickness, or diameter.	*When seasoned.* Length.		*When seasoned.* Breadth and thickness, or diameter.	*Weight in air of a cubic foot, oz. avoirdupois.* When first planed for experiment.	*When seasoned.*
			Ft.	*In.*	*Inches.*	*Ft.*	*In.*	*Inches.*	*Ounces.*	*Ounces.*
Pitch Pine Mast..	Virginia.........	Butt......	3	4	$18\frac{3}{8}$ diameter	3	4	$18\frac{1}{4}$ diameter	628	597
	North America....	Top......	7	6	10 by 8	7	6	$9\frac{15}{16}$ by $7\frac{15}{16}$	540	529
Yellow Pine Mast.	Canada..........	Butt......	2	4	18 by 18	2	4	$17\frac{7}{8}$ by $17\frac{7}{8}$	683	461
	North America....	Top......	5	0	16 by 16	5	0	$15\frac{7}{8}$ by $15\frac{7}{8}$	495	420
White Pine Mast...	American States.	Butt......	3	6	12 by 12	3	6	$11\frac{7}{8}$ by $11\frac{3}{4}$	555	405
		Top......	9	6	12 by 12	9	6	$11\frac{7}{8}$ by $11\frac{7}{8}$	633	448
Northern Pine Mast.	New York......	Butt......	3	6	$17\frac{5}{8}$ diameter	3	6	$17\frac{5}{8}$ diameter	658	549
		Top......	4	0	$7\frac{3}{4}$ by $7\frac{1}{4}$	4	0	$7\frac{11}{16}$ by $7\frac{7}{16}$	432	416
White Pine Mast...	New Brunswick.	Butt......	3	11	12 by 12	3	11	$11\frac{15}{16}$ by $11\frac{7}{8}$	679	436
		Top......	3	11	8 by 9	3	11	$7\frac{15}{16}$ by $8\frac{15}{16}$	411	368
Red Pine Mast.....	Canada........	Butt......	2	0	12 by 12	2	0	$11\frac{3}{4}$ by $11\frac{3}{4}$	672	569
		Top......	14	0	11 by 9	14	0	$10\frac{5}{8}$ by $8\frac{3}{4}$	570	503
Spruce Spar.......	Halifax........	Butt......	4	0	$8\frac{1}{2}$ diameter	4	0	$8\frac{3}{8}$ diameter	587	580
		Top......	4	0	$5\frac{1}{2}$ diameter	4	0	$5\frac{3}{8}$ diameter	541	554
Ditto............	Canada........	Butt......	4	0	7 diameter	4	0	$6\frac{15}{16}$ diameter	528	524
		Top......	4	0	$4\frac{1}{2}$ diameter	4	0	$4\frac{3}{8}$ diameter	485	512
Poon.............	East Indies.....	Butt......	4	0	17 diameter	4	0	$16\frac{3}{4}$ diameter	651	576
		Top......	6	0	9 by 9	6	0	9 by $8\frac{7}{8}$	771	695

TABLE—(*continued.*)

SPECIES (*in the language of commerce*).	*Country where produced.*	*What part of the Tree the pieces experimented on were cut from.*	DIMENSIONS.				*Weight in air of a cubic foot, oz. avoirdupois.*	
			When first planed for experiment.		*When seasoned.*			
			Length.	*Breadth and thickness, or diameter.*	*Length.*	*Breadth and thickness or diameter.*	*When first planed for experiment.*	*When seasoned.*
			Ft. In.	*Inches.*	*Ft. In.*	*Inches.*	*Ounces.*	*Ounces.*
Teak	East Indies	Butt	4 0	12 diameter	4 0	12 diameter	662	657
		Top	4 6	$6\frac{1}{2}$ by $6\frac{1}{2}$	4 6	$6\frac{1}{2}$ by $6\frac{1}{2}$	688	675
Yellow Wood	Cape of Good Hope	Butt	4 0	$5\frac{1}{2}$ diameter	4 0	$5\frac{7}{16}$ diameter	661	657
		Top	4 0	5 diameter	4 0	5 diameter	632	630
Stink Wood	Ditto	Uncertain	6 0	9 by 4	6 0	9 by 4	700	681
Letter Wood	Surinam	Butt	0 7	$4\frac{1}{4}$ by 4	0 7	$4\frac{1}{4}$ by 4	1286	1286
Cedar	Spanish America	Root	3 2	$5\frac{7}{8}$ by 6	3 2	$5\frac{7}{8}$ by 6	722	681
	Ditto	Trunk	2 0	12 by $5\frac{1}{2}$	2 0	12 by $5\frac{1}{2}$	457	453
Ditto	Canada	Uncertain	1 2	14 by 14	1 2	$13\frac{1}{4}$ by $13\frac{1}{4}$	909	753
Oak	England	Butt	1 10	12 by $11\frac{3}{4}$	1 10	$11\frac{5}{8}$ by $11\frac{3}{8}$	1113	743
Ditto		Top	2 0	$6\frac{1}{2}$ by 6	2 0	$6\frac{3}{16}$ by $5\frac{3}{4}$	1071	777
Elm	Ditto	Uncertain	4 0	11 by 11	4 0	$10\frac{3}{4}$ by $10\frac{3}{4}$	940	588
Bog Oak	Ireland	Ditto	0 $11\frac{3}{4}$	3 by $1\frac{1}{2}$	0 $11\frac{3}{4}$	3 by $1\frac{1}{2}$	1046	1046

9. To the same gentleman I am indebted for the following table relative to the loss of weight sustained by oak in seasoning. The eight pieces on which the experiments were made, were English oak, from 3 inches to $10\frac{3}{8}$ inches in thickness, and from 24 inches to 40 inches in length; the particulars of which are stated in the three upper lines in the following table; the dimensions there given being those of the pieces when first taken from the saw-pits in their rough state, viz. without planing; their not being originally cut for the purpose of these experiments, is the reason that most of them are found with dimensions partly fractional.

These several pieces were laid on the beams of a smith's shop, and placed at such a distance from the forges that the fire might only operate sufficiently to keep the air dry. They were converted from trees just received from the forest, and were weighed every month, from February 1810 to August 1812; at which latter period, as it was observed that the larger pieces lost but little of their weight, the weighing of them *monthly* was discontinued, and only performed annually, as shewn in the annexed table: from which it appears that the

Total weight, February 1810, was	$972\frac{1}{4}$ lbs.
Ditto, August 1815	$630\frac{1}{2}$
Weight lost........	$341\frac{3}{4}$

That is, more than one-third of the weight is lost in seasoning.

The specific gravity of No. 1, before seasoning, was 1074, and after that process only 720; and it is probable, that the specific gravity of oak is always within these limits; or, at least, that it seldom much exceeds the greatest, or falls below the least of these numbers.

TABLE OF EXPERIMENTS,

Relative to the Loss of Weight in Seasoning English Oak, *by* Mr. Couch.

	No. 1.	*No.* 2.	*No.* 3.	*No.* 4.	*No.* 5.	*No.* 6.	*No* 7.	*No.* 8.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
Length	24¾	25⅝	30⅝	31½	39¾	30½	37⅛	38¾
Breadth	16⅝	14⅛	16⅛	12⅛	16⅝	12¾	14¼	14¼
Depth	10⅝	9¾	8⅛	7⅛	6	5⅛	4	3
Periods of Weighing.	*lbs.*	*lbs.*	*lbs.*	*lbs.*	*lbs.*	*lbs.*	*lbs.*	*lbs.*
February 1810	163½	133	164	104½	163½	77¼	92	74½
March	154½	123¼	155¼	99	148¼	71⅛	[illegible]	65½
April	149½	118	151¼	96	142½	68¾	78	60¾
May	144½	113½	147	92¾	135¾	66¼	75	56¾
June	140½	109¼	143½	90¼	130¼	64	71¼	53¼
July	137¾	106¼	141	88¼	127	62	69½	51½
August	135¼	104½	139¼	87	123¾	61	67¾	50½
September	133	102¾	137¼	85¼	121	59¾	66	49¾
October	131¾	101½	136	84⅛	119¼	58¾	65	48¼
November	130¼	100¼	134¾	84	117¾	58¼	64½	47¾
December	129¾	99½	134¼	83¼	117½	58	63¾	47¾
January 1811	129	99	133¾	82¾	116¾	57¾	63½	47½
February*	130¼	100¼	135	84	118	57¾	65	47¾
March	127¼	98	132	81¾	115	57	62¾	47
April†	127¾	97	132¾	83	116	58½	64	46½
May	125¼	96½	130	80¼	113¼	56¼	61¾	46¼
June	124¼	95¼	129¼	79½	112¼	55¾	61	46¼
July‡	124½	96	129½	80¾	112½	55½	62¼	46
August	121¾	93¼	127	78½	109¾	54½	60¼	45½
September	122	92¼	127½	79¼	109	56	59½	45½
October	119¾	92	125½	77½	108¼	54	59½	45½
November	121	93¾	126½	77	110	56	59¾	45¾
December	119¾	91¾	125	77¼	108½	54	59½	45¾
January 1812	119	91¼	124½	76½	107¾	53¾	59	45¾
February	118¾	91¼	124	76¾	107½	53½	59	45¾
March	118½	91	124	76¾	107¾	53½	59¼	45¾
April	117½	90¾	123	76¼	106¾	53¼	58¾	45¾
May	117¼	90½	122¾	75¾	106½	53	58¼	45½
June	116¾	89¼	122	75½	106	52¾	58	45½
July	116	89¼	121½	75¼	105¾	52¾	58¼	45½
August 1812	115½	89	121	74¾	105¾	52½	58½	45¾
August 1813	111¼	85¾	116¾	72½	103¼	51½	57¾	45
August 1814	108½	85	114¼	71¼	103	51	57⅛	45⅛
August 1815	106¾	84½	112½	70½	102¾	51¼	57¼	45

* Very much rain since last weighed.

† Rained several days previous to weighing.

‡ Constant rain for two days previous to weighing.

The loss of weight in the preceding experiments was more rapid than in the similar experiments of Du Hamel: but much depends upon the nature of the soil in which the trees grow, as the timber of moist land loses more of its weight in seasoning, than that which is the produce of a drier and better soil.

10. The process of seasoning may be facilitated by boiling, steaming, &c. as appears from the following experiments of Mr. Hookey. The three pieces marked No. 1, 2, and 3, were English oak, each four feet long, and three inches square; all cut from the same timber. No. 1 was placed in the steam kiln for twelve hours; No. 2 was boiled for the same time in fresh water; and No. 3 was left in its natural state. The weights of the three pieces, previous to the experiment, and at the end of each month for half a year afterwards, were as stated below.

Times of Weighing.	*No. 1. Steamed.* Weight. lb. oz.	*No. 2. Boiled.* Weight. lb. oz.	*No. 3. Natural State.* Weight. lb. oz.
Previous to the experiment	16 12½	16 15	16 14
After ditto	16 6	16 14	16 14
June	15 1	15 10	16 5
July	14 2	14 12	15 14
August	13 13	14 0	15 5
September	12 10	13 6	15 0
October	12 5	12 10	14 12
November	11 10	12 5	14 8

Each of the pieces was placed in the same place, in the open air, and in the same position (i. e. verti-

cally), after the experiment, and were continued so during the six months that their weights were taken.

From the above, it appears that the process of seasoning went on more rapidly in the piece that was steamed than in that which was boiled; but that in the latter, the process was carried on much quicker than in the piece which was left in its natural state:

The first had its specific gravity reduced from 1050 to 744.
The second .... from 1084 to 788.
And the third .... from 1080 to 928.

We must look to the philosopher for a satisfactory solution of the problem presented in these results. Mr. Hookey* accounts for the facts by supposing, that the process of boiling or steaming dissolves the *pithy* substance contained in the air tubes, by which means the latter fluid circulates more freely, and that the seasoning thereby proceeds with greater rapidity.

11. From the several experiments above given, and from others found in Du Hamel's work above referred to, it appears,

1. That the density of the same species of timber, and in the same climate, but on different soils, will vary as much as in the ratio of seven to five; and that the strength of the same will be, both before and after seasoning, in nearly the ratio of five to four.

2. In healthy trees, or those which have not

* To this gentleman is due the ingenious idea of bending large ship timbers.—See Transactions of the Society of Arts, vol. xxxii.

already passed their prime, the density of the butt is in some cases to that of the top in about the ratio of four to three, and that of the centre to the circumference, as seven to five.

3. That the contrary has place if the tree is left standing after it has acquired full maturity; viz. the butt will in this case be specifically lighter than the top, and the centre than the outward part of the trunk within the bark.

4. That oak, in seasoning, loses at least one-third of its original weight; and this process is much facilitated by steaming or boiling.

On these subjects, as well as a variety of others, relative to the quality of timber, &c. which do not properly fall within the plan of this work, the reader is referred to the Treatise of Du Hamel above mentioned, where he will find much useful and important information.

Former Theories and Experiments relative to the Transverse Strength and Stress of Beams.

12. In the preceding chapter, it was only necessary to relate the results of such experiments as had been made with a view to ascertaining the direct cohesion of different woods; for as no combination of mechanical action has place in that operation, no particular theory is necessary to be established. There can be no doubt that the strength is in proportion to the number of ligneous fibres; and, therefore, *cæteris paribus,* to the area of the section of fracture. It is, however, very different in the case at present under consideration; where, instead of a direct application of the force in the line of the

fibres, it is only able to act upon them ultimately in that direction by the intervention of different mechanical agents, particularly the lever; and the efficacy of this force is again dependent upon the proportion between the compressibility and extensibility of the fibres, and the law which these two forces observe, according to the quantity of each; that is, according as the fibres are more or less extended or compressed; and this being the case, it is not surprising that different authors have assumed different hypotheses, and have thence produced contradictory results.

It is intended in this place to examine these several theories, and to give all the most important experiments that have been made relative to this particular strain.

13. Galileo, to whom the physical sciences are so much indebted, was the first who connected this subject with geometry, and endeavoured to compute the strength of different beams upon pure mathematical principles, by tracing the proportional strengths which different bodies possessed, as depending upon their length, breadth, depth, form, and position.

It appears that this philosopher was led to these investigations, in consequence of a visit which he made to the arsenal and dock-yards of Venice, and which were published in his Dialogues in 1633. He there considers solid bodies as being made up of numerous small fibres applied parallel to each other; and sought, or assumed, at first, the force with which they resisted the action of a power to separate them when applied parallel to their length;

and thence readily deduced, that their resistance in this direction was directly as the area of the transverse perpendicular section; that is, as the number of fibres of which the body is composed.

He next considered in what manner the same fibres would oppose a force applied perpendicularly to their length, and ultimately came to the following conclusion: "that when a beam is fixed solidly "in a horizontal position in a wall, or other im-"moveable mass, the resistance of the integrant "fibres is proportional to their sum, multiplied into "the distance of the centre of gravity of the area of "fracture from the lowest point."

14. In order to illustrate this theory a little more explicitly, let RSTV (*fig.* 1, *plate* 1,) represent a solid wall, or other immoveable mass, into which the beam CG is inserted, and let W be a weight suspended from its other extremity: then supposing the beam to be insuperably strong in every part except in the vertical section ABCD, the fracture must necessarily take place in that section only; and, *according to the hypothesis of this author,* it will turn about the line CD, whereby the fracture will commence in the line AB, and terminate in the former CD. Galileo also further supposes, that the fibres forming the several horizontal plates, or laminæ, from CD to AB, act with equal force in resisting the fracture, and therefore differ in their energy only as they act at a greater or less distance from the supposed quiescent line, or *centre of motion,* CD.

Now, from the known property of the lever, it is obvious, that the equal forces acting at the several

distances, $o1$, $o2$, $o3$, $o4$, &c. of the lever oe, will oppose resistances proportional to their respective distances; and therefore that their sum, that is, the constant force of each particle into its respective distance, is the force which must be overcome by the weight W, acting as on a lever, at the distance oK.

15. This will perhaps be better understood from the illustration given by M. Girard, in his "*Traité Analytique de la Resistance des Solides*," which is as follows: "Let ACIF, (*fig.* 2,) represent a longi-
"tudinal section of the beam CG, and w', w'', w''', &c.
"so many small equal weights passing over pins or
"pulleys, at r', r'', r''', r'''', &c. acting at the several
"distances, Cm', Cm'', Cm''', &c. each weight being
"supposed equal to the cohesion of its respective
"lamina; then denoting each of these weights by the
"constant quantity f, the sum of all their energies,
"or resistances, will be expressed by the formula:

$$Cm'.f + Cm''.f + Cm'''.f + Cm''''.f + \&c. =$$
$$f \times (Cm' + Cm'' + Cm''' + Cm'''' + \&c.)"$$

This, however, supposes the section ABCD, (*fig.* 1,) to be rectangular, or that the number of fibres in each horizontal lamina is the same. When the beam is triangular, cylindrical, or has any other than a rectangular section, the several small weights must be made proportional to the breadth of the section at the point where each is supposed to act: the illustration, however, is equally obvious.

Since, then, the whole resistance to fracture is

made up of the sum of the resistance of every particle or fibre, acting at different distances on the lever CA, which is supposed to turn upon C as a fulcrum, there must necessarily be some point in that lever, in which, if all the several forces were united, their reaction to the weight W would be exactly the same as in the actual operation; *and this point is the centre of gravity of the section represented by* AC.

For let ABC, (*fig.* 3,) represent the section of any formed beam whatever, FH, any variable absciss, $= x$, and DE, the corresponding double ordinate, $= y$; then, by what is stated above, the energy or force of all the particles in the line DE will be as DE × HF, or as $x\,y$; and consequently, the fluxion of that force will be $y\,x\,\dot{x}$, and the sum of all these forces will, therefore, be denoted by $\int y\,x\,\dot{x}$. Now the area of the section may be expressed by $\int y\,\dot{x}$; and, assuming G as the centre of energy sought, we shall have

$$FG \times \int y\,\dot{x} = \int y\,x\,\dot{x}$$

$$\text{Whence } FG = \frac{\int y\,x\,\dot{x}}{\int y\,\dot{x}}$$

which is the well-known expression for the centre of gravity.

16. From these considerations, or at least from others tantamount to them, Galileo deduces his general theorem for the resistances of solids; which, from what is above stated, is obviously as follows: viz.

When a beam is solidly fixed with one end in a wall, or other immoveable body, the weight neces sary to produce the fracture, is to the force of direct cohesion of all its fibres, as the distance of the centre of gravity of the section of fracture, from the lowest point of that section, to the length of the beam, or the distance at which the weight acts from the same point.

From other investigations, which it is unnecessary to exhibit in this place, the author endeavours to shew, that whatever weight is sufficient to break a beam, fixed as above, double that weight will be necessary to break a beam of equal breadth and depth, and of twice the length, when supported at each end on two props; and four times the same weight, when the latter is fixed with each end solidly in a wall: and from these general principles flow a number of corollaries and deductions, which, notwithstanding they are not (as will be shewn) *all* founded in fact, nor agree with actual experiment, yet they are too curious and interesting, and are too generally received as correct, to be passed over unnoticed. We select the following abstract of them from "*Gregory's Mechanics,*" vol. i. p. 108 *et seq.*

1. The strength of a beam or bar, supported at both ends to resist a fracture by a force acting laterally, is as a solid made by a section of the beam, in the place where the force is applied, into the distance of its centre of gravity, from the point or line where the breach will end.

2. In square beams, the lateral strengths are as the cubes of the breadths or depths.

3. In cylindrical beams, the lateral strengths are as the cubes of their diameters.

4. In rectangular beams, the lateral strengths are conjointly as the breadths and squares of the depths.

5. The lateral strength of such a beam, with its narrower face upwards, is to its strength with the broader face upwards, as the breadth of the latter to the breadth of the former.

6. When the beam is fixed with one end in a wall, and the fracture is caused by a weight suspended at the other end, the same corollaries will still obtain.

7. Cylinders and square prisms have their lateral strengths proportional to the cubes of their diameters or depths directly, and their lengths inversely.

8. Similar prisms and cylinders have their strengths proportional to the squares of their like linear dimensions.

9. The lateral strength of two cylinders (of the same matter) of equal weight and length, one of which is hollow, and the other solid, are to each other as the diameters of their ends.

To these we may add the following, as flowing immediately from the same principles.

10. The strength of a square beam, when placed with its diagonal vertically, is to the strength of the same beam with its sides vertical, as the diagonal to the side, or as $\sqrt{2}$ to 1.

11. The strength of a cylinder is to that of the greatest square prism that can be cut out of it, placed with its diagonal vertically, as the area of a circle to that of its inscribed square.

12. When a triangular beam is supported at both ends, its strength, when the edge of the beam is uppermost, is to its strength, when the base, or opposite side, is uppermost, as 2 is to 1.

17. Nothing can be desired more simple and elegant than the above, as a general theory; but, unfortunately, it is founded on hypotheses, which have nothing equivalent to them in nature. In the first place, it assumes the beam to be inflexible, and insuperably strong, except at the section of fracture: secondly, that the fibres are inextensible and incompressible: and, thirdly, that the beam turns about its lowest point when fixed at one end, or its upper when supported at both, and therefore that every fibre in the section is exerting its force in resisting extension: and, lastly, if this be not implied in the former objection, that every fibre acts with equal energy, whatever may be the tension to which it is exposed. Now, with regard to the first of these suppositions, it is obvious that no beam of timber, or any other body with which we are acquainted, is perfectly inflexible; nor any (and more particularly timber) whose fibres are not both extensible and compressible; and, consequently, a beam of such matter will not turn about its lowest point, as a fulcrum: and, lastly, the supposition of every fibre exerting a constant resistance, independently of its quantity of extension, if it be apparently correct, it is of that nature which ought not to be assumed without being first verified by experiment.

18. The theory of Galileo having these radical defects, it necessarily happened, as soon as it was

attempted to compare its results with experiments, (which the author himself had never done,) that it was found defective. The first person, we believe, who did this, was Mariotte, a member of the French Academy, who, having soon discovered its inaccuracy, proposed to substitute another theory in its place, which was published in 1680, in his "*Traité du Mouvement des Eaux;*" and here we find the first notice of extensible and compressible parts of the section of fracture, the neutral axis, &c. This attracted the attention of Leibnitz, who, after examining the theory of Galileo and the experiments of Mariotte, published his own thoughts on the subject, in a memoir which appeared in the Leipsic Acts, in 1684; but he rather retrograded than advanced the science.

19. He stated, that every body before breaking was subject to a certain degree of deflection, which could not have place if the fibres were, as Galileo had supposed, inextensible; and thence, assuming the principle first suggested by Dr. Hooke, viz. *ut tensio sic vis*, he concluded that every fibre, instead of acting with an equal force, exerted a power of resistance proportional to its quantity of extension; or, which is the same, proportional to its distance from the line about which the beam was supposed to turn: but he still considered the fibres to be incompressible, or at least, what amounts to the same, that the beam turned about its lowest or highest point, accordingly as it was fixed at one end, or supported at both.

20. Thus, to use a similar illustration in this case

to that we have done in the former, instead of the fracture being opposed by the action of the equal forces or weights $\overset{\prime}{w}$, $\overset{\prime\prime}{w}$, $\overset{\prime\prime\prime}{w}$, &c. (*fig.* 2,) the resistance is supposed to be equal to the decreasing weights $\overset{\prime}{w}$, $\overset{\prime\prime}{w}$, $\overset{\prime\prime\prime}{w}$, &c. (*fig.* 4,) these being to each other in the proportion of their respective distances from the axis of rotation.

The only alteration which this hypothesis introduced into the final results, was the removal of the centre of energy G, to another point I, (*fig.* 3,) nearer or farther from the centre of motion, according to the figure of the transverse section of the beam: and this new point is found to be distant from that axis, *by a quantity equal to the product of the distances of the centres of gravity and oscillation from the axis of motion, divided by the depth of the section.*

For, let ABC (*fig.* 3,) represent, as before, the area of fracture of any beam, FH $= x$ any variable absciss, and DE $= y$, the corresponding double ordinate. Also, make CF $= d$, and let f represent the absolute and ultimate force of a fibre at C, at the instant of rupture: then, since the resistance opposed by each fibre is supposed to vary as its tension, or as its distance from F, we have $d : x : : f : \frac{f\,x}{d} =$ the force of a particle at H; and the number of particles acting at this distance being y, we shall have $\frac{f\,x\,y}{d}$, for the sum of the resistances of all the fibres or particles in the line DE: but this force, acting upon the lever at the distance HF, its resistance will be expressed by $\frac{f\,x^2\,y}{d}$; and hence the

sum of all the resistances of every fibre in the section, will be $= \int \frac{f y x^2 \dot{x}}{d}$.

Now this is to be equal to the direct cohesion of all the fibres acting at some required distance FI; that is,

$$\mathrm{FI} \times \int y \dot{x} \times f = \frac{f}{d} \times \int y x^2 \dot{x}, \text{ or}$$

$$\mathrm{FI} = \frac{1}{d} \times \frac{\int y x^2 \dot{x}}{\int y \dot{x}}$$

The variable part of this expression is exactly equivalent to the general formula $\frac{\int y x^2 \dot{x}}{\int y x \dot{x}}$, (for the centre of oscillation of a surface,) multiplied by $\frac{\int y x \dot{x}}{\int y \dot{x}}$ (the well-known expression for the centre of gravity), that is, using $\frac{1}{d}$ as a coefficient,

$$\mathrm{FI} = \frac{1}{d} \times \frac{\int y x^2 \dot{x}}{\int y \dot{x}} = \frac{\int y x \dot{x}}{\int y \dot{x}} \times \frac{\int y x^2 \dot{x}}{\int y x \dot{x}} \times \frac{1}{d}$$

as is obvious. And since these centres are generally known in most of the figures which fall under consideration in the present inquiry, we may avail ourselves of them, independent of calculation, in determining what may properly be termed the *centre of energy*, or *centre of tension:* but, in other cases, recourse must be had to the general fluxional expression

$$\mathrm{FI} = \frac{1}{d} \times \frac{\int y x^2 \dot{x}}{\int y \dot{x}}$$

21. Referring again to the formula previously found for the centre of energy on the Galilean hypothesis, and denoting the absolute strength of cohesion on a square inch by f; also writing d for the depth of the beam in inches, a the area of fracture, and l the length likewise in inches: then the general expression for the ultimate strength of any beam, fixed with one end in a wall, would be on the hypothesis of

$$\text{Galileo,} \quad S = \frac{\int y x \dot{x}}{\int y \dot{x}} \times \frac{a f}{l}$$

$$\text{Leibnitz,} \quad S = \frac{\int y x^2 \dot{x}}{\int y \dot{x}} \times \frac{a f}{l}$$

When the beam is *supported* at both ends, these must be each multiplied by four; and when *fixed* at both ends, by eight.

This being the case, both theories give the same results, so far as relates to the comparison of similar-formed beams, but of different dimensions: thus, from both it appears, that when the breadth and depth are the same, the strength varies inversely as the length: when the length and depth are the same, the strength varies directly as the breadth; and when the length and breadth are the same, the strength varies as the square of the depth: deductions which have been found to agree very nearly with experiment.

22. There are other conditions, however, resulting from the same formulæ, in which the two theories

are totally irreconcilable with each other, and in which neither will agree with actual experiment.

In the first place, although the proportions are the same, the absolute strength in the one case is to that in the other as two to three in rectangular beams; and in triangular ones the disagreement is still more striking. According to Galileo, the strength of a triangular beam with its edge upwards, when fixed by one end in a wall, or with its base upwards, when supported at both ends, is to the strength of the same beam in the reversed position, as one to two; and according to Leibnitz, as one to three: whereas we have found, from various experiments, that in some woods, (fir, for instance,) the strength is greater in the former position than in the latter, in the ratio of more than seven to six; but varying in different woods, according to their comparative resistance to compression and extension.

Again, square beams fixed, in one instance with the side vertical, and in the other with the diagonal vertical, have their strengths, according to Galileo, in the ratio of 1 to $\surd 2$; and according to Leibnitz, in that of $\frac{4}{3} : \surd 2$; whereas experiments shew the beam to be *stronger in the former position than in the latter.*

In both theories, also, the strength of hollow cylinders, not bored through the axis, but nearer one side than the other, vary according as the boring is nearer the upper or lower surface, and is greatest of all when the cylinder is infinitely thin on that side about which it is supposed to turn; whereas experiment shews the very reverse of this,

and that the beam is absolutely weakest, when, according to both these writers, it ought to be the strongest.

It is a very remarkable fact, that, notwithstanding numerous experiments have been made by different authors, and under a variety of circumstances, not one (at least that I am acquainted with) has pointed out these discrepancies between the theory and practice. Most of them have stated that the ultimate strength, as found by experiment, did not accord with the theoretical result; but in no instance have the striking anomalies above referred to been clearly stated, or even hinted at, by these experimentalists.*

23. James Bernoulli, however, although it does not appear that he was aware of the great defect in these theories, was still fully convinced that they were not correct; and he, therefore, after examining the theory of Leibnitz, undertook to investigate the question again *de novo*.

* I ought here to mention one exception, which has occurred while these experiments have been in progress, viz. an anonymous writer in "*The Philosophical Magazine*," who stated, that he had found the triangular beam of the same strength, whether the base or edge was uppermost; and that in square beams the strength was less with the diagonal vertical than when placed in the usual position.

I have also been favoured with the unpublished experiments of Mr. Couch, where the same defect of the former theories is pointed out.

Also, since the first edition of this work was published, a highly valuable treatise on the Principles of Carpentry, by Mr. Thomas Tredgold, has appeared, in which these anomalies are clearly and satisfactorily stated.

This celebrated mathematician observed, that the instant a body is broken across with a transverse strain, such as we have been considering, a part of the fibres only are in a state of tension, and a part in a state of compression; a circumstance that had not before been introduced into the conditions of this problem, except perhaps by Mariotte; and he moreover doubted of the justness of the principle, "*ut tensio sic vis*," employed by Leibnitz, and made some experiments, whereby he proved that, at least, this is not a universal law of nature. But he unfortunately stopped at this point, contenting himself with shewing the inadequacy of the theory he had been examining; but without substituting any new one in its place, except so far as his theory of the *elastic curve* (a problem which arose out of the present question,) may be considered as applicable to this subject. Had he pursued the idea he seems first to have promulgated, of a part of the fibres being stretched and a part compressed, and consequently, that the line about which the beam turns is somewhere within the area of the section of fracture, we might have expected, from his extraordinary talents, a complete solution of this interesting problem: instead of which, he contented himself with stating a few general observations, and with pointing out the difficulty of determining the neutral axis, or of that line which suffers neither compression nor extension; which is the principal desideratum for establishing a correct theory.

I have not had an opportunity of consulting the original Memoir of Bernoulli, which is published in the volume of the French Academy of Sciences for 1704; but, from an abstract given of it by Girard,

in his "*Traité Analytique de la Résistance des Solides,*" p. 7, it appears that this author has demonstrated, "that whatever be the position of the axis of equilibrium, or neutral axis, the same force, applied to the same arm of a lever, is always capable of producing the same effect, whether all the integrant fibres be extended or compressed about this axis, or whether only a part of them be extended, and a part compressed."

As I have not, as above stated, been able to examine the original investigation, I am unwilling to offer any critical remarks on a deduction said to have been drawn from demonstration by so celebrated an author as James Bernoulli: but it is obvious that he could not arrive legitimately at such a conclusion, without assuming the resistance to extension, and the resistance to compression, to be equal to each other; a limitation which he certainly could only introduce as an hypothesis.

24. The theories of Euler and Lagrange follow next in order of time; but as these are not made to depend upon any particular law of resistance, nor even on the force of direct cohesion, but on what these authors denominate the *relative* and *absolute elasticity* of bodies, they may be more conveniently considered in the section which treats of the force of pression: in which place also it will be best to examine M. Girard's results and experiments. It only remains, therefore, to offer a few remarks relative to the theory and observations of Dr. Robison, as they are given, under the article STRENGTH, in the Encyclopædia Britannica, above referred to.

25. This is the only theory, of any importance, in which the position of the neutral axis, or that line in a beam which suffers neither extension nor compression, is introduced as a necessary datum; and even here only, as it were, *en passant*, the principal investigations being founded on the hypothesis of Leibnitz. The necessity, however, of considering the situation of this line in the investigations is clearly pointed out; but as no advance is made with regard to assigning its position, nor any step made towards determining the ratio between the resistance to tension, as compared with that of compression, the formulæ which this author has deduced from these considerations are not in a state to be submitted to actual experiments. The principle upon which it is founded, however, will be readily understood. Dr. Robison considers the beam as turning on the neutral axis, or that line on its section which is neither extended nor compressed; but he still does not treat the operation of breaking as compound, but as a simple mechanical process, tending towards the fracture of the fibres submitted to tension; and only considers the compressed fibres as acting as a fulcrum, the virtual axis of which is the centre of compression. "It is plain," says this author, "and the remark is important, that this last centre of effort is the real fulcrum of the lever, although A is the point where there is neither extension nor contraction; for the lever is supported in the same manner as if the repulsion of the whole line were exerted at that point." And in another place he observes: "Thus we see that the compressibility of bodies has a great influence on their

power of withstanding a transverse strain. We see that in the most favourable supposition of equal dilations and compressions, the strength is reduced to only one-half of the value of what it would have been, had the body been incompressible. This is by no means obvious; for it does not readily appear how compressibility, which does not diminish the cohesion of a single particle or fibre, should impair the strength of the whole. The reason, however, is sufficiently convincing when pointed out. In the instant of fracture, a smaller portion of the section is actually exerting cohesive forces, while a part of it is only serving as a fulcrum to the lever by whose means the strain on the section is produced." This theory will be examined more at length in a subsequent chapter.

The above are the principal theories that have been given relative to the transverse strain and strength of timber, in which the law of resistance is a necessary datum.* Much, however, has been written on this subject, which the writer has not had the means of consulting; viz. a Memoir, by *Bulfinger*, in the "*Comment. Petropolitan.*" 1729; a Memoir of *Varignon*, published in the volume of the Academy of Sciences for 1702; and another, by *Parent*, in the same Transactions, for 1708: *Coulomb* has also a paper on this subject in the "*Mem. par les Sçavans Etranges,*" tom. vii.; and since the publication of the first edition of this work, an ingenious paper has been published in the 4th vol. of the

* We may also refer the reader to Tredgold's "Principles of Carpentry," where a very judicious view is taken of different theories connected with this inquiry.

Manchester Memoirs, by Mr. Hodgkinson; besides several others of less note.

Former Experiments on the Transverse Strength of Beams.

26. It is remarkable, that scarcely any of the authors who have offered new theories relative to the strain we have been considering, have themselves performed any experiments: this, at least, seems to have been the case with Galileo, Leibnitz, James Bernoulli, Euler, Lagrange, and Dr. Robison: the latter, it is true, mentions, in here and there a place, some isolated experiments, but he has nowhere given the results, nor employed them as the foundation of any hypothesis: and the others have not, we believe, attempted any. It is also equally singular, that those authors (of which there are many) who have carried on experiments, have commonly had in view the theory of some preceding writer, and have generally contented themselves with stating the results, and commonly the discrepancies between them and the theoretical deductions, but without attempting to establish any thing more accurate to supply the place of those hypotheses, which they have found to be defective.

27. It has been before observed, that Mariotte was the first who called the attention of mathematicians to the defect in the theory of Galileo; before which time, no doubt seems to have been entertained of its accurate agreement with experiment. This author was induced to undertake his experiments with a view to proportion the thickness of

water-pipes to the height of the water they had to support; but they were made on so limited a scale, and on pieces of such small dimensions, that, notwithstanding they indicated the inadequacy of the Galilean hypothesis, they were not calculated to furnish data whence a more accurate one could be deduced. In fact, the only result of these experiments which is necessary to be mentioned in this place, is the author's own deductions relative to glass rods. He found that rods of this substance, when fixed at both ends, bore double what they would do when only supported on two props; which so far agrees with the hitherto received hypothesis in the comparison of these two strains; but which, however, has never been found to obtain on bars or beams of timber, except by Musschenbroeck.

28. *M. Parent*, in "Les Mémoires de l'Académie des Sciences" for 1707 and 1708, gives an account of the experiments he made on small prisms of wood; and states, that having successively fixed them first at one end, then *supported* them at both, and lastly *fixed* them at both, their strengths in these positions were respectively as the numbers 4, 6, and 9; the two latter class of bars being each double the length of the former, so as to leave the lever of the same length in all three; and *Belidor** afterwards made similar experiments on battens of even-grained sound oak, and found similar results; at least as far as regard those beams which were supported and fixed at their extremities, as appears from the following tablet: where the reader will

* "Science des Ingénieurs," 1729.

observe, that the numbers are given, agreeably to the author's own statement, in French inches and pounds; which may, however, be readily converted into English measure and weight by the following proportions; viz.

76 : 81 : : 1 inch *Eng.* : 1 inch *French.*
100 : 108 : : 1 lb. *avoird.* : 1 lb. *French.*

BELIDOR'S *Experiments on Oak Bars, in French Inches and Pounds.*

No. of Experiments.	*Length in Inches.*	*Depth in Inches.*	*Breadth in Inches.*	*Weight in lbs.*	*Mean Weight.*	*Remarks.*
1	18	1	1	400 415 405	406	Supported at both ends.
2	18	1	1	600 600 624	608	Ends firmly fixed.
3	18	1	2	810 795 812	805	Ends supported.
4	18	2	1	1570 1580 1590	1580	Ends supported.
5	36	1	1	185 195 180	187	Ends supported.
6	36	1	1	285 280 285	283	Ends firmly fixed.
7	36	2	2	1550 1620 1585	1585	Ends supported.
8	36	$2\frac{1}{3}$	$1\frac{2}{3}$	1665 1675 1640	1660	Ends supported.

29. Here, by comparing experiments Nos. 1 and 3, the strength appears to be proportional to the breadth.

Nos. 3 and 4 shew the strength proportional to the square of the depth.

Nos. 1 and 5 shew the strength to be nearly in the inverse ratio of the length, with a deficiency in the longer pieces.

Nos. 5 and 7 shew the strength to be nearly proportional to the breadth and square of the depth, or as the area into the depth.

Experiments 1 and 2, and those of 5 and 6, shew the increase of strength, by fixing the ends, to be in the proportion of 2 to 3, or as 6 to 9, as found by Parent.

30. The latter coincidence in the results of these two authors would naturally, one would suppose, have induced subsequent theorists to examine the hypothesis, which gave the ratio of 1 to 2, in order to discover what modification it might require to make it agree with experiment; instead of which, however, they have contented themselves with attributing it to want of accuracy, or any thing rather than to a defect in the theory: thus, Dr. Robison says, "The theory gives the proportion of 2 to 4; but a difference in the manner of fixing may produce this deviation." It will be seen, however, as we proceed, that there is no reason to suspect any inaccuracy in the experiments, but that the ratio which these authors have found, and which has been confirmed by other experiments, is very nearly or exactly such as the theory requires. I am aware, in making this assertion, that the experiments per-

formed by Musschenbroeck are at variance with those of the authors above quoted, and that they even approach towards a coincidence with the former theory. It is difficult to explain this anomaly, and I am by no means disposed to doubt the accuracy of Musschenbroeck's statements. The only observation, therefore, I shall allow myself to offer, is, that the pieces on which the experiments were made being very small, the proportional weights might not be so distinctly marked as in larger bars: in our experiments the pieces submitted to this test were of larger dimension, and the fixing perfectly secure, (as will be seen in the subsequent description of the apparatus); and the results, with a slight modification, agree very nearly with those of *Parent* and *Belidor*.

It would be useless to pass in review here, the numerous experiments that have been made by different authors, and given in various parts of their respective works, as many of these are not only at variance with one another, but with themselves; and from which, therefore, little or no useful information is to be obtained.

31. This, however, is not the case with those of *MM. Buffon* and *Du Hamel:* these two philosophers were directed by the French Government to make experiments on this subject, and were supplied with ample funds and apparatus for carrying them into effect. The results of their experiments are found in "Les Mémoires de l'Académie des Sciences" for 1738, 1740, 1741, 1742, and 1768; and many interesting observations in Du Hamel's treatises, "*Sur l'Exploitation des Arbres;*" and "*Sur la Conservation et le Transport des Bois.*" These are

the most extensive and valuable series of experiments that have yet been made on oak-beams, and contain the most important practical information, particularly the last of M. Buffon; his first having been attempted, like those of most of his predecessors, on pieces of too small dimensions, to draw from them any conclusive and satisfactory results. Amongst other curious experiments related by Du Hamel, he mentions the following:

He took 16 bars of willow, 2 feet long and half an inch square, and supporting them on props under the ends, he loaded them by weights hung on the middle. He broke four of them by weights of 40, 41, 47, and 52 pounds; the mean being 45; and having then cut four of them $\frac{1}{3}$ through, on the upper side, and filled up the cut with a thin piece of hard wood, stuck in pretty tight, they were broken by 48, 54, 50, and 52 pounds; the mean of which is 51. He then cut four others $\frac{1}{2}$ through, and they were broken by 47, 49, 50, and 46; the mean being 48. The four remaining bars were cut $\frac{2}{3}$ through, and their mean strength was 42 pounds.

In another set of experiments, he took six battens of willow, 36 inches long and $1\frac{1}{2}$ inch square; the medium strength of which was 525 pounds.

Six others of the same dimensions being cut $\frac{1}{3}$ through, and the saw-cut filled up as above, broke at a medium with 551 pounds.

Six others cut $\frac{1}{2}$ through required 542 pounds; and six others cut $\frac{3}{4}$ through required 530 pounds.

In another experiment, a bar of the same dimensions as those above was cut $\frac{3}{4}$ through; and after being loaded for some time, and till nearly broken,

it was then taken down, and the thin slip replaced with a wedge-like piece to fill up the cut, so as to straighten the bar again; it then bore 577 pounds; and we have repeated similar experiments on fir-beams, with nearly the same results.

32. Buffon's experiments are the next in order of time, and the most important of any that have yet been made; both as regard the number of them, and the size of the pieces: many of them having been from 20 to 28 feet in length, and from 4 to 8 inches square. This philosopher, as before observed, was furnished by the French Government with ample funds, and every necessary means for carrying on his experiments on a grand scale; and he discharged the duty thus imposed upon him in a manner highly creditable to himself, and to the satisfaction of the Academy; but he did not, perhaps, possess the mathematical knowledge necessary for making the best use of his results. His experiments, however, are not the less valuable; as they are no doubt faithfully related, and furnish the best foundation that has yet been given for the establishment of a correct theory.

He commenced his operations, with Du Hamel, on pieces of small dimensions; and tried them in succession from the heart to the bark of the tree, and from the root upwards. From these experiments it was found, that the heart was the densest, that the density decreased from hence to the circumference, and that the strength decreased also in nearly the same proportion.

He also made trial of the proportional strength of battens, accordingly as they were laid, with the

annual layers, vertical or horizontal. Let, for instance, *fig.* 4, *plate* 3, represent the annual layers of a large piece of timber, and *abcd*, and ABCD, the section of two equal battens, cut at equal distances from the centre, there will be a difference in the strength of these two pieces nearly in the ratio of 7 to 8, the section ABCD being the strongest, supposing them to be broken in the positions in which they are represented in the figure: and this difference arises from the cohesion of the layers with each other being considerably less than that between the fibres themselves. I have attempted, by some experiments, to shew the quantity of this lateral cohesion, although it must be allowed to be rather a subject of curiosity than utility; for large beams, whose strength it is the most important to be acquainted with, commonly occupy the whole, or nearly the whole section of the tree.

M. Buffon found likewise, that oak-timber lost much of its strength in the course of drying, or seasoning; and therefore, in order to secure uniformity, his trees were all felled in the same season of the year, were squared the day after, and experimented on the third day. Trying them in this green state, gave him an opportunity of observing a very curious phenomenon; namely, that when the weights were laid briskly on, nearly sufficient to break the log, a very sensible smoke was observed to issue from the two ends, with a sharp hissing noise, which continued all the time the tree was bending or cracking.

This philosopher, as above stated, drew no important conclusions from his experiments: he seems to have had in view no favourite theory, either of his

own or of any other writer, and was therefore free from any bias, or any desire to accommodate his experiments to a particular hypothesis: besides, his beams were too large for him to deceive himself in this respect, as there is reason to believe has been the case with some authors. Upon the whole, these are certainly the most valuable experiments that have yet been made upon the transverse strength and strain of oak timber; whether they be considered as the means of furnishing practical precedent, or theoretical data: the following table of this author's results will therefore, it is presumed, be acceptable to the English engineer, for whose convenience the several results are reduced to English weights and measures.*

* In converting the metres and kilogrammes into English inches and avoirdupois pounds, the utmost accuracy has not been attempted; the following proportions having been thought sufficiently approximative for this purpose; viz. instead of taking the kilogramme as equal to 2 lbs. 3 oz. 0·73 drs. it has been supposed equal to $2\frac{1}{5}$ lb. or 2 lb. 3 oz. $3\frac{1}{5}$ drs., by which means the reduction was much more readily effected, while the error is too small to be of any importance in gross results of this kind. And with similar views the metre was assumed equal to 39·6 inches; which is = 40 inches *minus* ·4 inches: whence, multiplying the metres by 40, and deducting the sum from the product, after removing the decimal point two places, will give the number of inches: thus,

$$\begin{array}{r} \cdot1082 \text{ metres} \\ 40 \\ \hline 4\cdot3280 \\ \cdot0432 \\ \hline 4\cdot2848 \text{ inches} = \cdot1082 \text{ metres;} \end{array}$$

and so on for the other dimensions.

TABLE

Of the Results of BUFFON'S *Experiments on the Transverse Strength of Square Oak-beams.*

No.	*Side of Square.* in inches.	*Side of Square.* in metres.	*Length.* in feet and inches.	*Length.* in metres.	*Weight of the pieces.* in lbs.	*Weight of the pieces.* in kilogr.	*Weights which broke the pieces.* in lbs.	*Weights which broke the pieces.* in kilogr.	*Deflection before cracking.* in inches.	*Deflection before cracking.* in metres.
1			7 6	2·2732	64·56	29·34	5756	2616	3·47	·0946
2			7 6	2·2732	60·25	27·39	5676	2580	4·82	·1217
3			8 6·8	2·5979	73·17	33·26	4950	2250	4·01	·1013
4			8 6·8	2·5979	67·79	30·81	4842	2201	5·00	·1262
5	4.28	·1082	9 7·7	2·9227	83·85	37·66	4401	2005	5·17	·1307
6			9 7·7	2·9227	76·39	34·73	4250	1932	5·89	·1488
7.			10 8·5	3·2473	90·37	41·09	3900	1773	6·25	·1578
8			10 8·5	3·2473	88·23	40·11	3884	1760	6·96	·1758
9			12 10·3	3·8969	107·60	48·91	3281	1491	7·50	·1894
10			12 10·3	3·8969	105·45	47·93	3174	1430	7·50	·1894
11			7 6	2·2732	101·14	45·98	12670	5759	2·67	·0676
12			7 6	2·2732	95·32	43·33	12133	5515	2·67	·0676
13			8 6·8	2·5979	111·91	50·87	10653	4842	2·85	·0721
14			8 6·8	2·5979	109·76	49·89	10411	4732	3·12	·0789
15			9 7·7	2·9227	126·97	57·72	9038	4108	3·21	·0811
16			9 7·7	2·9227	124·82	56·74	8958	4072	3·51	·0878
17			9 7·7	2·9227	173·75	56·25	8822	4011	3·75	·0946
18			10 8·5	3·2473	142·04	64·56	7774	3534	3·39	·0856
19			10 8·5	3·2473	139·91	63·60	7586	3448	3·74	·0946
20			10 8·5	3·2473	138·28	62·85	7639	3472	4·28	·1082
21			12 10·3	3·8969	167·87	76·30	6510	2959	5·89	·1488
22	5·35	·1353	12 10·3	3·8969	165·71	75·32	6563	2983	6·16	·1556
23			15 0	4·5464	191·54	87·06	5811	2641	8·57	·2164
24			15 0	4·5464	189·38	86·09	5595	2543	8·83	·2231
25			17 1·7	5·1959	224·90	102·23	4861	2164	8·65	·2186
26			17 1·7	5·1959	220·59	100·27	4600	2091	8·74	·2209
27			19 3·4	5·8453	249·65	113·48	4034	1834	8·57	·2164
28			19 3·4	5·8453	248·57	113·00	3248	1799	8·74	·2209
29			21 5·1	6·4947	284·12	129·64	3523	1601	9·46	·2389
30			21 5·1	6·4946	278·71	126·69	3416	1553	10·71	·2706
31			25 8·6	7·7939	333·59	151·63	2367	1076	11·88	·2976
32			25 8·6	7·7939	330·36	150·16	2286	1039	12·05	·3652
33			30 0	9·0928	391·69	178·04	1936	880	19·78	·4870
34			30 0	9·0928	365·40	166·09	1882	855	23·57	·5952

TABLE—(*continued.*)

No.	*Side of Square.* in inches.	*Side of Square.* in metres.	*Length.* in feet and inches.	*Length.* in metres.	*Weight of the pieces.* in lbs.	*Weight of the pieces.* in kilogr.	*Weights which broke the pieces.* in lbs.	*Weights which broke the pieces.* in kilogr.	*Deflection before cracking.* in inches.	*Deflection before cracking.* in metres.
35			7 6	2·2732	138·27	62·85	20715	9416		
36			7 6	2·2732	129·66	58·93	20069	9122		
37			8 6·8	2·5979	160·33	72·88	16894	7679	2·50	·0631
38			8 6·8	2·5979	157·11	71·41	16517	7508	2·58	·0653
39			9 7·7	2·9227	178·63	81·19	14473	6579	2·67	·0676
40			9 7·7	2·9227	177·02	80·46	13607	6285	2·83	·0766
41			10 8·5	3·2473	202·30	91·96	12347	5612	3·21	·0811
42			10 8·5	3·2473	200·18	91·00	11863	5392	3·74	·0946
43	6·43	·1624	12 10·3	3·8969	241·04	109·56	9900	4500	4·28	·1082
44			12 10·3	3·8969	237·82	108·10	9684	4402	4·38	·1107
45			15 0	4·5464	274·40	124·73	8016	3644	4·82	·1217
46			15 0	4·5464	273·33	124·24	8070	3668	4·38	·1107
47			17 1·7	5·1959	316·37	143·80	6725	3057	5·89	·1488
48			17 1·7	5·1959	315·29	143·32	6967	3167	6·25	·1578
49			19 3·4	5·8453	359·42	163·37	6052	2751	7·94	·2006
50			19 3·4	5·8453	156·18	161·90	5918	2690	9·10	·2299
51			21 5·1	6·4946	405·69	184·40	5406	2457	10·17	·2570
52			21 5·1	6·4946	403·53	183·43	5246	2384	9·46	·2389
53			8 6·8	2·5979	219·52	100·00	28140	12791	2·94	·0743
54			8 6·8	2·5979	219·52	100·00	27926	12693	2·68	·0676
55			9 7·7	2·9227	244·28	111·03	24535	11152	3·31	·0836
56			9 7·7	2·9227	242·12	110·05	23562	10712	3·12	·0789
57			10 8·5	3·2473	273·33	124·24	21145	9611	2·73	·0689
58			10 8·5	3·2473	271·15	123·25	20769	9440	3·21	·0811
59			12 10·3	3·8969	324·98	147·72	18078	8217	3·12	·0789
60			12 10·3	3·8969	323·90	147·23	16733	7606	3·56	·0900
61	7·5	·1894	15 0	4·5464	382·01	173·64	14634	6652	4·46	·1127
62			15 0	4·6464	377·72	171·69	13828	6285	4·01	·1013
63			17 1·7	5·1959	436·90	198·59	11944	5429	5·17	·1307
64			17 1·7	5·1959	433·67	197·12	11729	5331	5·62	·1420
65			19 3·4	5·8453	488·55	222·07	10163	4622	5·89	·1488
66			19 3·4	5·8453	468·55	222·07	10113	4597	6·25	·1578
67			21 5·1	6·4946	653·45	247·01	9200	4182	8·39	·2119
68			21 5·1	6·4946	654·55	247·50	8608	3913	9·10	·2299
69			10 8·5	3·2473	356·19	161·90	29916	13598	3·21	·0811
70			10 8·5	3·2473	356·19	161·90	28709	13049	2·41	·0608
71			12 10·3	3·8969	427·22	194·19	24619	11190	3·21	·0811
72			12 10·3	3·8969	425·60	193·45	23654	10750	3·12	·0789
73			15 0	4·5464	496·08	225·49	21575	9807	4·10	·1036
74	8·57	·2165	15 0	4·5464	493·93	224·51	20894	9538	3·39	·0856
75			17 1·7	5·1959	564·96	266·80	18078	8217	5·53	·1398
76			17 1·7	5·1959	563·88	266·31	17163	7801	4·01	·1013
77			19 3·4	5·8453	639·21	290·55	14526	6603	4·82	·1217
78			19 3·4	5·8453	638·53	290·06	13801	6309	4·37	·1104
79			21 5·1	6·4946	712·34	323·79	12670	5759	6·96	·1758
80			21 5·1	6·4946	710·23	322·83	13128	5967	6·43	·1623

33. It has been observed, that the preceding table may be considered as furnishing the most useful results relative to the transverse strength of oak-beams, of any hitherto made public; both as they regard practical precedent and theoretical data: but, with reference to the former, the engineer must bear well in mind the green state of the wood when the experiments were performed, which adds much to its strength, on account of the fibres in that state offering a much greater resistance to compression, than when the timber has been well dried and seasoned.

This may not, perhaps, appear obvious to those who have not well considered the mechanism of the transverse strain; nor can it be properly explained, till that subject has been treated of more at length in a subsequent chapter. The phenomenon, however, observed by Buffon, as mentioned above, is sufficient to shew that this must be the case: the juices which issued from the ends of the timber, were pressed through the pores by the action of the weight; and the latter, therefore, must necessarily be greater, to produce the same degree of deflection, than if no such juices existed, as is the case in well-seasoned timber; or they are at least in much smaller quantity, and therefore less resistance is in this case offered to compression: in consequence of which, the neutral axis is brought lower, and fewer fibres are exerting their force of tension to resist fracture.

The foregoing, it is presumed, is an abstract of all the most important experiments that have hitherto been published relative to this strain: but I have been favoured with some from different quarters,

not hitherto published, which it will be proper to give in this place.

34. A knowledge of the strength and elasticity of timber being subjects of the highest importance in the constructions of ships, &c. the surveyors of His Majesty's navy have, at different times, ordered experiments to be made, directed to this object; and they have in the most handsome manner supplied me with every information they were in possession of, relative to those inquiries; a favour for which I am equally indebted to the liberal views of those gentlemen, and to the friendly interference and recommendation of John Knowles, Esq. secretary to that board, through whom it was solicited.

The following table contains the results of experiments carried on in His Majesty's dock-yard at Deptford, by Colonel Beaufoy, on English and Dantzic oak, Riga fir, and pitch pine. The several pieces were each 5 feet long and 2 inches square, fixed at one end in a mortice to the length of one foot, so that the part projecting was 4 feet; and the weight was hung on at that distance from the fulcrum. The 25 pieces of Dantzic oak were cut from the same tree, of which the mean specific gravity was 854. The several pieces of Riga fir were also all from one tree, of which the mean specific gravity was 537; as were those of pitch pine, but the specific gravity is not stated. Of the English oak, the first 6 pieces were from one tree, of which the specific gravity was 922, and the other 13 from another, the latter very irregular and cross-gained, but its weight is not given: nor do I find any indication of the particular weight of each piece, nor the

situation it occupied with regard to its distance from the heart or centre. It is simply stated, that the last piece of oak was the heart of the tree, and that it was the weakest.

The deflections were measured in degrees and minutes, on a graduated arc of the same radius as the beam, viz. 4 feet, and were taken as every 14 pounds were put on: we have given, however, only the mean, the last weights, and the corresponding deflections. It appears from all these experiments, that the deflections are very nearly in the ratio of the weights, till about one-half, or a little less than one-half the weight is laid on, after which they become more rapid, and very irregular; from which it may be concluded, that a piece of timber will safely support about half the weight that is necessary for producing its ultimate rupture.*

* Colonel Beaufoy has recently published his experiments at full length, in Thomson's "Annals of Philosophy;" and as I have, in the above table, taken the ultimate weight, instead of the mean, given by the author himself, it may be proper to state my reasons for so doing.

In the first place, I have preferred the ultimate results, because these experiments are thus more readily compared with my own, and with those of other experimentalists; and, secondly, because the mean given by Colonel Beaufoy is not correctly deduced. The experiments were made by loading the scale successively with 7 lb. weights, and the mean given by the author is found by adding these several weights together, viz. 7 + 14 + 21 + 28 + &c., to the breaking weight, and then dividing the sum by the number of the terms. This is obviously the same as adding 3½ lbs. to half the last weight, for the mean weight; for which process no particular reason is assigned; nor can any, that I am aware of, be offered. This is not, however, the principal defect; for, in some instances, the weights are not regularly increased by 7 lbs. at a time; but towards the conclusion of an experiment, a succession of weights of

2 lbs. each is sometimes added: but in these cases also the mean weight is still found, by dividing the sum of all the weights by the number of them; which evidently destroys all proportionality between the several ultimate and mean weights; a comparison which doubtless ought to be kept in view in all cases. In other respects, the experiments appear to have been made with great care; and, from the number of them, they will be esteemed of considerable value.

TABLE *of Experiments carried on in his Majesty's Dock-yard, Deptford, on Beams of different Woods, fixed at one End: by* COL. BEAUFOY.

No of Experiment.	Dantsic Oak, 25 Pieces, 4 ft. long 2 inches square.		Deflection in Degrees, &c.	Riga Fir, 25 pieces, 4 ft. long, 2 inch. square.		Deflection in Degrees, &c.	Pitch Pine, 24 pieces, 4 ft. long, 2 inches square.		Deflection in Degrees, &c.	English Oak, 19 pieces, 4 feet long, 2 inch. square.		Deflection in Degrees, &c.
	lbs.		° ′	lbs.		° ′	lbs.		° ′	lbs.		° ′
1	98		2 3	98		1 24	98		1 15	98		1 15
		196	6 12		182			287	7 0		266	
2	98		2 6	98		1 21	98		1 18	98		1 14
		193	7 0		175	5 6		266	6 24		273	6 24
3	98		2 21	98		1 14	98		1 20	98		1 12
		175			182	4 42		280	5 36		224	
4	98		2 36	98		1 12	98		1 6	98		1 10
		168	7 12		182	5 48		257	4 50		284	7 0
5	98		2 48	98		1 23	98		1 8	98		1 19
		151	5 54		238	5 0		270	5 30		231	5 0
6	98		2 32	98		1 26	98		1 6	98		1 14
		161	6 12		168	3 0		274	6 0		273	6 50
7	98		2 24	98		1 36	98		1 0	98		1 17
		175	7 12		182	4 6		294	6 0		245	
8	98		2 9	98		1 25	98		1 6	98		1 16
		184	6 15		203	5 30		266	6 30		238	
9	98		1 54	98		1 20	98		1 20	98		1 20
		192	6 30		238	6 0		245	7 30		238	
10	98		1 46	98		1 16	98		1 24	98		1 32
		193	5 25		259	7 12		203	5 30		238	
11	98		1 58	98		1 26	98		1 10	98		1 47
		183			217	5 30		274	6 0		224	
12	98		1 51	98		1 39	98		1 8	98		1 26
		202	9 0		168	3 50		274	7 42		231	
13	98		2 24	98		1 15	98		1 0	98		1 32
		154	6 0		154	4 0		326	7 0		189	
14	98		2 4	98		1 26	98		1 15	98		1 20
		175			182	4 30		280	5 30		245	
15	98		2 23	98		1 21	98		1 10	98		1 28
		140	3 54		210	4 30		287	6 30		231	
16	98		2 37	98		1 14	98		1 15	98		1 44
		112	2 54		252	6 12		256	7 30		231	
17	98		2 1	98		1 20	98		2 12	98		1 34
		174	6 30		189	3 36		182	6 30		238	
18	98		1 57	98		1 30	98		1 12	98		1 30
		179	6 12		161	4 0		277	8 30		210	
19	98		1 40	98		1 38	98		0 54	98		1 46
		214			154	4 36		308	5 30		182	
20	98		1 54	98		1 31	98		1 2			
		133			238	5 0		301	7 30			
21	98		2 19	98		1 20	98		1 8			
		161	7 0		224	6 12		301	7 30			
22	98		1 57	98		1 18	98		1 12			
		161	5 24		238	4 48		224	5 0			
23	98		2 36	98		1 16	98		1 12			
		147	5 30		175	5 48		252	5 30			
24	98		2 30	98		1 24	98		0 54			
		133	4 24		245	3 0		336	7 0			
25	98		3 18	98		1 16						
		130	5 48		231							

1st Six,
Sum 1551.
Mean 258 lbs.

13 *following*,
Sum 2740.
Mean 211 lbs.

35. These experiments furnish us with the absolute and comparative strength of the four following woods; viz.

Length 4 feet, 2 inches square	Dantzic oak..	167 lbs.....Sp. gr. 854	
	Riga fir*	202 lbs.....Sp. gr. 537	
	Pitch pine...	272 lbs.....Sp. gr.	
	English oak ..	258 lbs.....Sp. gr. 922	
	Ditto	211 lbs.	

Other experiments were made by the same gentleman on battens of $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, and 3 inch square, and of the same length. The particulars are not stated, but it appeared from them that the ratio of the strengths a little exceeded the ratio of the cubes of the sides.

36. Other experiments were also made upon pieces of the same dimensions, spliced and fixed in different ways: the *scarph* in all of them was 12 inches long, and 13 inches from the end, viz. about an inch from the fulcrum: the results were as follow:

Scarph up and down ..	No. 1. broke in the splice	112 lb.
	No. 2. ditto...........	109 lb.
Scarph flatwise, large end uppermost, and towards the fulcrum	No. 1. nails drew through the small end of the scarph............	104 lb.
	No. 2. ditto...........	98 lb.
Scarph flatwise, small end towards the fulcrum............	No. 1. broke in the thick part of the scarph...	84 lb.
	No. 2. ditto...........	90 lb.

* It may be proper to observe, that No. 13, in Colonel Beaufoy's Report of the Riga fir, was very irregular, having been broken with

From these experiments it is inferred, that the two former positions of spliced pieces are preferable to the last.

Experiments by John Peake, Esq. *and* M. Barrallier.*

37. It is necessary, in order that the reader may properly understand the results contained in the fourth, fifth, and sixth columns of the following table, to explain the nature of the apparatus by which these several pieces were submitted to experiment. This is shewn in *fig.* 11, *plate* 2, where RSTV represents the vertical section of an oak pillar, 12 inches square, having a hole of 2 inches square, for the purpose of receiving the end of the batten, ABCD; the pillar itself being securely fixed, between the principal floor-joist and the tye-beam, in the mould-loft in Woolwich dock-yard; and BF is a semicircular piece of oak, of 6 inches radius, well fixed to the principal pillar, to prevent the batten from crippling at B. This semicircle was divided into inches and parts, and as the weights were successively applied at W, the batten was deflected, and in some measure bent over BF; and the numbers in the columns above mentioned shew to what extent this bending took place.

only 98 lbs.; this experiment is therefore rejected, and its place is supplied with experiment No. 26. It may also be farther stated, that the above mean weights are obtained by dividing the sum of all the breaking weights by the number of them.

* This gentleman was for several years a resident in this country, but was formerly, and is, I believe, at present, principal builder in the dock-yard at Toulon.

As to the numbers in the other columns, they will be readily understood, from the description given at their heads in the table; the first shewing the number of the experiments; the second, the number of years the pieces had been in store; the third, the specific gravity; the fourth and fifth, the part of the arc, BF, which came in contact with the batten, with 56 lbs. and 112 lbs. respectively; the sixth, the contact which remained after removing the last weight; the seventh column shews the whole curvature; the eighth, the weight under which the piece crippled; the ninth the weight under which it broke; and the tenth contains sundry remarks.

TABLE I.

Of Experiments on Riga Fir, Battens two Inches square, fixed at one End, and the Weight acting at five Feet from the Fulcrum.

Note.—These pieces were all kept dry.

No. of Experiment.	Years in Store.	Specific Gravity.	Arc received by the battens under the weight of 56 lbs.	Arc received by the battens under the weight of 112 lbs.	Arc remaining after the last weight was removed.	Total curvature.	Weight under which the beam crippled.	Weight under which it broke.	REMARKS.
			inches.	inches.	inches.	inches.	lbs.	lbs.	
1	13	474	3¾	7½	1	12	112	144	Part of a jib-boom.
2	6	693	3¾	6½	1	16	202	220	
3	13	474	4¼	7½	1	12	112	144	Same as No. 1.
4	13	513	3¼	5¼	0¼	13	167	194	Ditto.
5	6	768	4⅜	12½	2⅛		112	112	Sound batten.
6	6	804	3¼	7	0¾		126	129	Ditto.
7	6	756	3¼	7	0½		126	127	Ditto.
8	6	696	3⅞	7¼	0½		133	141	Ditto.
9	6	720	3⅞	7½	0⅜		126	126	Broke short.
10	6	726	4	7⅞	0½		137	137	Ditto.
11	6	756					77	77	Very shaky.
12	6	726	3⅜	8¼	0½		126	126	Sound.
13	6	720	2¾	5⅜	0¼		127	138	Ditto.
14	6	720	3	5¾	0⅜		147	147	Ditto.
15	6	708	2⅞	6	0⅜		147	147	Ditto.
16	6	522	3⅞	7⅝	0⅝		133	135	Very dry.
17	10	558	3¾	7¾	0¾		133	133	Broke short.
18	10	564	3½	6¾	0½		140	149	Ditto.
19	10	522	3	5¾	0¼		140	140	Fine texture.
20	8	546	3⅛	6¼			133	140	Ditto.
21	8	558	2⅜	5⅜	0⅜	13½	140	147	Broke short.
22	3	828	3⅝	6¾	0⅞		161	161	Ditto.
23	6	693	3¾	6½	1	16	202	220	Same as No. 2.
24	6	705	3¾	6¾	0½	13¾	168	182	Broke sudden.
25	13	486	3¾	7½	1	12	112	116	
26	10	513	3¼	5¼	0¼	13¾	167	194	
27	8	546	3½	5⅜	0	15	168	202	
28	8	561	2½	4¾	0¾	9¾	168	191	
		27)17856 Mean, 633	Sum, rejecting No. 11.					27)4132 Mean, 153	

The preceding table, by Colonel Beaufoy, reckoning the strength to be inversely as the length, gives 5 : 4 : : 202 : 161 lbs. for the mean; which is in defect only 1 lb. The mean from Table I. and II. being 162 lbs. at 4 feet.

TABLE II.

Experiments in every respect similar to the last, except that the several Pieces were kept wet.

No. of Experiment.	*Years in Store.*	*Specific Gravity.*	*Arc received by the battens under the weight of* 56 *lbs.*	112 *lbs.*	*Arc remaining after the last weight was removed.*	*Total Curvature.*	*Weight under which the beam crippled.*	*Weight under which it broke.*	*REMARKS.*
			inches.	inches.	inches.	inches.	lbs.	lbs.	
1	29	639	3	5	0	12½	193	207	
2	6	615	2¾	4½	0⅜	13½	248	261	
3	13	534	3¼	6	0¼	14	126	158	
4	13	555	2¾	5	0	15	153	208	
5	29	639	3	5	0	12½	193	207	
6	6	876	2¾	5½	1⅛		136	136	Very shaky.
7	6	666	2⅛	4⅛	0⅛		140	199	Sound.
8	6	666	2⅛	4⅛	0⅛		158	190	Ditto.
9	6	696	2⅛	4¼	0⅛		154	172	Ditto.
10	6	762	2⅜	4⅝	0 1/16		168	180	Ditto.
11	6	690	2½	4¾	0 1/10		168	168	Little shaky.
12	6	720	2	3⅝	0⅛		168	203	Very sound.
13	6	690	2	3¾	0⅛		176	186	Sound.
14	6	708	2⅝	4½	0⅛		128	128	Very cross-grained.
15	6	726	2⅛	3¾	0⅛		209	214	Sound.
16	6	702	2⅛	4	0¼		214	214	Ditto. [grained.
17	10	606	4¼	10¼	1⅛		133	133	Very shaky, and cross-
18	10	720	3¼	12	1		112	112	} These broke very slowly
19	10	642	2⅝	5⅝	0¼		159	159	}
20	10	666	2⅞	6⅜	0⅜		123	132	Ditto, shaky.
21	10	540	3½	8¼	0¾		117	117	Coarse-grained.
22	10	528	3⅝	9⅛	0¾		132	132	Cross-grained.
23	8	648	4⅛	12			112	112	Coarse-grained.
24	8	552	3¼	6⅞	0⅝		151	153	
25	29	738	2¼	4¾	0¼	13	146	160	
26	10	684	3½	10⅜			112	135	
27	10	684		12⅛	1¼			137	
28	6	615	2¾	4½	0⅜	13½	248	261	
29	10	492	4¾	9	1⅛	13½	128	135	Very shaky.
30	6	594	2¾	4¾	0½	12½	224	233	
31	8	564	4¼	7½	1	13½	140	161	
32	13	534	3¼	6	0¼	14	126	158	Coarse soft grain.
33	13	495	4¾	11½	1½	16½	112	149	Shaky and knotty.
34	29	639	3	5	0	12½	197	207	
35	8	600	3¼	5¾	0¼	14	168	199	
36	13	510	3½	5	0½	15	147	147	
37	13	555	2¾	5		15	153	208	

Sum, 37)23390

Mean, 632 wet.
Mean, 633 dry.
Mean, 537 of both.

3)1802

Mean, 600

Sum, 37)6371

Mean, 175 wet.
Mean, 153 dry.
Mean, 161 Colonel Beaufoy.

3)486

Mean of the three, 162

TABLE III.

Containing similar Experiments on Battens of the same Dimensions, of different Kinds of Wood.

No. of Experiment.	Years in Store.	Specific Gravity.	Arc received by the battens under the weight of 56 lbs.	Arc received by the battens under the weight of 112 lbs.	Arc remaining after the last weight was removed.	Total curvature.	Weight under which the beam crippled.	Weight under which it broke.	REMARKS.
					Virginia Yellow Pine.				
			inches.	inches.	inches.	inches.	lbs.	lbs	
1	Time unknown.	564	4¾	...	...	10	98	98	Dry and defective.
2	do.	720	2⅜	4⅜	0⅜	16½	246	251	Ditto.
3	do.	498	6	...	...	15¼	233	233	Ditto.
4	do.	618	4¾	3½	0⅜	26¾	206	234	Ditto.
5	do.	498	3¼	6⅝	0⅜	...	126	135	Part of old topmast.
6	do.	522	3⅞	8⅜	0⅞	11⅜	133	133	Dry.
7	do.	492	3½	6½	0⅝	...	140	147	Ditto.
					Pitch Pine.				
8	do.	816	2	3¾	0¼	9¼	196	203	Dry.
9	do.	816	1¼	2⅝	0⅛	...	336	365	Ditto.
10	do.	996	2½	3⅝	0¼	12½	238	244	From Lukin's kiln.
11	do.	738	2	4	0⅛	12½	224	332	Dry.
12	do.	732	2	3¼	0⅛	11¾	308	308	Ditto.
13	do.	696	2¼	3⅞	0⅛	14	287	303	Ditto.
14	do.	708	2½	4½	0⅛	17	273	293	Ditto.
15	do.	720	2¼	4¼	0½	...	140	...	Defective.
					Canadian White Pine.				
16	1	648	4¼	..	...	14	98	123	Wet.
17	10	672	4⅝	...	...	14	98	119	Ditto.
18	8	714	4	...	...	14	84	103	Ditto.
19	8	660	5¾	...	...	14	84	108	Ditto.
20	4	720	3¾	...	...	14	84	91	Ditto.
21	4	714	3¾	...	...	10⅛	84	96	Ditto.
22	8	618	3⅝	10⅝	1⅜	18¼	116	122	Dry.
					Larch.				
23	4	526	7⅜	16⅛	4	34	...	170	Dry.
24	4	540	3⅛	7½	0⅞	14⅞	133	133	Ditto.
25	4	570	5⅛	10½	1	15	...	137	Ditto.
26	4	526	3⅜	6⅜	0¾	16¼	160	162	Ditto.
					Dantzic Fir.				
27	4	690	2⅜	4¾	0⅛	...	158	158	Wet.
28	4	648	2½	4¾	0¼	12½	140	140	Ditto.
29	4	630	2½	4¾	0⅛	12½	140	140	Ditto.
30	3	624	3¾	6¼	0¾	11¾	186	192	Ditto.
					Ash.				
31	1	858	2	4⅛	0⅜	16	224	239	Quite green.
32	1	828	2⅛	4⅞	0⅝	18⅛	...	217	Ditto.
33	...	660	3⅞	6⅛	0½	12¾	...	196	Old capstan bar.
					Teak.				
34	2	672	2⅜	4¾	....	16½	224	271	Old bowsprit.
35	2	606	2¾	4¼	...	12½	224	257	Old bowsprit.

38. The preceding table furnishes the following means; viz. each bar being

5 feet long, and 2 inches square.	Riga	Dry 153 / Wet 172	162	mean sp. gr. 633
	Virginia yellow pine		189	558
	Pitch pine		256	777
	Canadian white pine		109	678
	Larch		150	540
	Dantzic ditto		156	648
	Ash		217	782
	Teak		264	639

It may be remarked, that the strength of pitch pine, according to these experiments, exceeds very considerably what was found by Colonel Beaufoy; while that of the Riga fir, taking a mean between the wet and dry, is exactly the same in both: but it is to be observed, that in the experiments by Messrs. Peake and Barrallier, the bending of the pieces over the arc, as above described, shortens the ultimate radius, and therefore they ought to be stronger than with the uniform radius of 5 feet; consequently, the specimens of Riga fir in these experiments were really weaker than those of Colonel Beaufoy, although they apparently agree with each other.

Mr. Couch's *Experiments on Triangular Oak-Beams, &c.*

39. In a preceding chapter, I have given the detail of several valuable experiments by Mr. Couch, of His Majesty's dock-yard, Plymouth; and the two following tables I owe to the same gentleman. They exhibit the detail and results of experiments on the lateral or transverse strength of triangular prisms of

Canadian oak, the sections of which were equilateral triangles, the sides being 3 inches: and also, on some pieces reduced to the form of trapezoids, by cutting off the vertex, or upper angle, to one-third of the depth.

The short pieces, viz. those 3 feet 3 inches, Table I. were fixed, by one end horizontally, in a 3 inch mortise; the others, as given in Table II., which were 6 feet 6 inches, were fixed at each end into 3 inch mortises, so as to prevent the ends from rising; and in both cases they were so well fitted, as to require small blows of the mallet to drive them in.

These experiments were made in order to obtain data connected with mast-making, and to ascertain how far the commonly received notion was correct; namely, that if the vertex, or upper edge of a triangular prismatic beam, be cut off to one-third of the depth, the pieces will be stronger than before; or, in other words, that a part opposes more resistance than the whole: which assertion, as anticipated, was satisfactorily contradicted by the following results.

These experiments are also very conclusive on another point; viz. that the strength of triangular prisms does not follow the law laid down, either by Leibnitz or Galileo; for, according to the former, the weights required to break a beam of this kind, with its base upwards, ought to be three times greater than in the reverse position; and according to the latter, it ought to be double. Now, the mean of the first seven experiments is 306, and of the next four 348; which is very far from the weight required in either of the above theories: and it will be seen that on fir, the latter position is weaker than the former.

TABLE I.

Experiments on Triangular Oak-beams, by* Mr. B. Couch. *Pieces 3 Feet 3 Inches long, fixed by one End horizontally into a Pillar; 3 Feet beyond the Prop.

Weight placed on the end.

Order of the Experiments.	*Position, form, &c.*	*Deflection below the first position.*	*Weight in lbs. supported.*	*Weight of the pieces.* lb.	oz.	*The same pieces placed end for end, after altering their position, or form.*	*Deflection below the first position.*	*Weight in lbs. supported.*	*Weight of the pieces.* lb.	oz.
1		9	290	3	1					
2		9	313	3	3½					
3		9	290	3	3		9	261	2	13
4	Angle upward.	9	333	3	6		9	271	2	15½
5		9	309	3	6	Reduced to Trapezoids, narrow end upward.	11	248	2	15½
6		9	308	3	5					
7		10	298	3	4		9	270	2	15
8		16	332	3	10					
9	Angle downward.	11	349	3	7	Angle upward.	9	286	3	7
10		11	351	3	3					
11		11	360	3	4					
12	Trapezoid, narrow end up.	8	283	3	4					
13		11	285	3	1½					

Sum of first seven weights = 2141
Sum of the next four = 1392

7)2141 4)1392

306 mean 348 mean.

Sum of six trapezoids = 1618

6)1618

269 mean.

TABLE II.

Experiments; by* Mr. Couch, *on Pieces* 6 *Ft.* 6 *In. long, each End fixed into Pillars horizontally*—6 *Feet between the Props.

Weights placed on the Middle.

Order of the Experiments.	*Position, form, &c.*	*Deflection below the first position.*	*Weight in lbs. supported.*	*Weight of pieces.* lb.	oz.	*REMARKS.*
1	Angle upward.	6	980	7	5	
2		6	896	6	9	
3		6	1008	7	3	
4		5	1116	6	14	
5		6	1288	6	15	
6	Angle downward.	3	1056	6	14	Fractured $\frac{3}{8}$ inch on the angle.
		4	1166			Ditto 1 inch on the angle.
		7	1257			Broke nearly off.
7		2	870	7	2	Sprung a little on the angle.
		3	947			Broke nearly oft.
8		3	1003	7	3	Sprung $\frac{1}{2}$ inch on angle, and continued breaking with the addition of every $\frac{1}{2}$ cwt., fibre after fibre, $\frac{3}{8}$ inch at a time, till all gave way.
		5	1366			
9		$2\frac{1}{2}$	1285	7	14	Sprung, without giving warning, from angle to half the depth.
10		$2\frac{1}{2}$	1395	9	2	Sprung $\frac{1}{2}$ an inch on angle.
		$3\frac{1}{2}$	1686			
11	Trapezoid, narrow face upward.	6	1319	8	6	Coarse, strong grain.
12		7	1099	6	0	Fine, weak grain.

40. I ought to add, that I have been favoured by Mr. Harvey of Plymouth, a mathematician of great talent and application, with the result of similar experiments, which he made with a particular object, and which agree very nearly with the above.

41. Table III. exhibits the detail and results of experiments carried on also by Mr. Couch, on the lateral or transverse strength of Riga, Norway, and Halifax spars; as also on pieces of timber wrought to the shape of the said spars, (viz. frustrum of cones,) converted from large logs of red pine, yellow pine, and oak, all the growth of Canada.

The spars and other pieces were all of the same dimension, viz. 27 feet long, $3\frac{1}{4}$ inches diameter at the butt, and to the distance of 5 feet from the butt: the upper end was $1\frac{1}{4}$ inch in diameter.

They were fixed by the greater end horizontally into a mortise, the prop or fulcrum being 5 feet from the butt; and the weights were placed one foot from the smaller end, leaving a lever or purchase of 21 feet.

TABLE III.

Experiments on Riga, Norway, and Halifax Spars, Red and Yellow Pine, &c., by Mr. Couch.

Order of Experiments.	*Species of Wood.*	*Weight of each Spar.*	*Deflection.*	*Weights which broke them.*	*REMARKS.*
		lbs.	*feet.*		
1	Riga spar	$29\frac{1}{4}$	11	130	
2	Riga spar	$29\frac{1}{2}$	11	137	Upset or compressed,
		...	$12\frac{3}{4}$	144	very much broke.
3	Norway spar	32	12	168	Upset, lower part.
		...	$13\frac{3}{4}$	172	
4	Norway spar	$36\frac{1}{2}$	11	180	Upset, very much.
		...	$12\frac{3}{4}$	206	
5	Halifax spar..........	$37\frac{3}{4}$	$11\frac{1}{4}$	115	
6	Halifax spar..........	$34\frac{1}{2}$	$12\frac{3}{4}$	188	The tension of the fibres
7	Red pine timber......	$40\frac{1}{4}$	14	150	of this spar was much
8	Red pine timber......	$42\frac{3}{4}$	14	180	increased by being
9	Red pine timber......	$42\frac{1}{4}$	14	202	placed near a large
10	Yellow pine timber.	$33\frac{1}{4}$	rapid deflection	56	fire for several days. Fibres undulated.
11	Yellow pine timber.	32	14	146	
12	Yellow pine timber.	$33\frac{1}{2}$	rapid deflection	56	Fibres undulated.
13	Oak timber....	$52\frac{1}{2}$	16	231	
14	Oak timber...........	$53\frac{1}{2}$	18	254	

The experiments which have been now detailed relative to the transverse strains, are, it is presumed, all that are deserving of any particular notice in this place; we shall therefore now proceed to state some of the principal theories and experiments, connected with the pressure of columns, and the absolute and relative elasticity of the fibres, when acted on in a direction parallel to their length.

Of the principal Theories and Experiments on the Pressure and longitudinal Resistance of Columns.

42. The several theories referred to above, are all founded upon the most simple and obvious mecha-

nical principles, are therefore easily illustrated, and readily comprehended even by those who have made but little advance in mathematical sciences: their disagreement with each other is, however, a sufficient proof that the hypotheses on which they rest are (at least some of them) defective and erroneous. The subject on which we are now about to enter is of a very different kind, as it involves considerations connected with the highest and most difficult branches of the modern analysis, and, we may add, some paradoxes, which have required all the genius of a Euler and a Lagrange to unravel. The instruments, if we may be allowed the expression, have been too delicate to operate successfully upon the materials to which they have been applied; so that while they exhibit, under the strongest point of view, the immense resources of analysis, and the transcendent talents of their authors, they unfortunately furnish but little, very little, useful practical information: yet this historical chapter would undoubtedly be left incomplete, if we did not attempt to give the reader some idea of the principles on which this theory is founded.

43. Reference has already been made to the memoirs of James Bernoulli on the elastic curve; and it will be necessary again to refer to the same, in order to trace from their source the elegant investigations of MM. Euler and Lagrange on what these authors term the *absolute* and *relative elasticities* of wood and other materials.

Bernoulli having observed, that flexible bodies assume a certain curve, when strained by any external force, he imagined, that this curvature

ought to depend upon their elasticity; and he therefore undertook to investigate the properties of the curve which a perfectly elastic lamina would take, in attempting to bring its two extremities into contact; and he succeeded in obtaining its equation,* which is transcendental. The investigation of this author might be applied with facility to every case in which the force was exerted in a direction perpendicular, or in any way inclined to the direction of the lamina; but it failed when the force was supposed to act in a direction parallel to its length, viz. when the two ends were pressed with forces directly opposed to each other.

44. The problem remained in this state till Euler undertook its solution, in the Appendix to his well-known work, "*Methodus inveniendi Lineas Curvas*," &c. where he demonstrates, that the resistance in this case is as the absolute elasticity, directly; and inversely, as the square of the length; the solids being supposed prismatic or cylindrical: but that, when the lamina was infinitely thin, its resistance, whatever might be its length, would be infinitely great; a paradox which appeared almost or entirely inexplicable.

When a body, considered as an aggregate of parallel fibres, is pressed in the direction of its length, it is difficult to conceive how any inflection can have place, since no reason can be assigned why it should bend in one part more than in another; still the theory indicated that this inflection ought to take place under a determinate weight, while any

* See "Mémoires de l'Académie," 1705.

force less than this would produce no effect whatever: this again, which appeared to destroy the uniformity of the law of continuity, presented another anomaly, which the author undertook to explain in a memoir on the resistance of columns, published in the Berlin Acts for 1767.

Euler, as above stated, (allowing his hypothesis,) demonstrated that the weight under which a column begins to bend is directly as its absolute elasticity or moment of elasticity; and this ought to be deduced from the equation of the curve: but this equation not being integrable, or, in other words, having no finite fluent, the finite value of the above element cannot hence be found; but, on the other hand, as this is wholly independent of the curvature of the solid, we may suppose the latter so small, that the element of the curve shall differ insensibly from that of the absciss; that is, they may be considered equal to each other; by which means, the radius of curvature is exhibited by a very simple expression, and the equation is thus converted into another, easy to integrate, and from which is readily determined the weight necessary to produce the inflection.

45. Lagrange found, (in the "Memoirs of Berlin," for 1769,) for the resistance of an elastic lamina pressed parallel to its length, precisely the same expression as that which Euler had before deduced from his investigation; and he now undertook to ascertain, whether the practice of enlarging columns at about a third of their height, followed in some instances by architects according to the principles of Vitruvius, had any tendency to increase their

strength. He sought at first the curve according to which they began to bend, and found that there were no limits to the number of its points of inflection, provided only that the weight was properly assigned.* And submitting afterwards his equation to a more rigorous examination, he demonstrated, that the greatest ordinate is imaginary, while the weight with which the columns is loaded has not attained a certain limit; which therefore explains, on principles purely analytical, the difficulty which Euler, in the memoir above stated, had already endeavoured to account for, without resting it, however, on the basis of demonstration.

This difficulty being thus surmounted, Lagrange sought first which of the solids, generated by the revolution of the conic sections, would offer the greatest resistance, parallel to its elementary fibres, its height and volume remaining the same; and found that it was the conoid generated by a right line, that possessed this property; and that of all columns having the form of a truncated cone, that which has its two bases equal, (viz. the cylinder,) has the greatest resistance. Whence, and from a similar result drawn from a general investigation on the principles of the calculus of variations, the author concludes, that if the enlargement of columns improves the elegance of their appearance, it adds nothing to their strength; and that, neglecting their own weight, the cylinder is the most eligible form.

46. It is only necessary to add to the above, that Euler, in 1778, again recurred to his investigations

* "Memoirs of Turin," 1770—1773.

on this subject; and as he had before considered the columns void of gravity, he now undertook, in three memoirs, (which were the last efforts of this celebrated geometer,) to estimate the influence of their own weight in causing their inflection. In the first, he concluded that the column would not bend under its own weight, except when its height was infinite; which, however, being obviously contradictory to some of his former deductions, led him to revise his investigation; and by introducing certain horizontal forces before neglected, he found in his second memoir, that the height necessary to produce an inflection fell within certain limits: the determination of which limits forms the subject of the third memoir, above referred to.* A highly philosophical examination of this interesting and difficult branch of the theory of the strength of materials, is also given by Dr. Young, in the first volume of his "Lectures on Natural Philosophy," but like those to which we have last referred, it is not supported by practical experiments. Mr. Tredgold, however, in his "Elementary Principles of Carpentry," and in his "Essay on the Strength of Cast Iron," two highly valuable works for those engaged in architectural and mechanical constructions, has reduced the theoretical views of the author first named to practical cases; and to these works we must beg to refer the reader for information.

47. It remains now to give some details relative to the theory and experiments of M. Girard, as

* Acta Petro. 1778.

published by that author in his "*Traité Analytique de la Résistance des Solides.*"

This work is divided into four sections, and an introduction; to which latter we have been indebted for many of the preceding historical statements and remarks.

The first section contains the investigation of general formulæ of resistance for beams acted upon by a transverse strain, with their application to the hypotheses of Galileo and Leibnitz, &c. &c. This application presents the means of comparing the theories of these two authors, and consequently of pointing out the difference in the results in a variety of cases; but nothing is said with the purpose of shewing which of them should be preferred, or whether they are not both erroneous: the reader is, therefore, left to make his own choice, without any thing to guide him in his determination. The investigations relative to the strains upon beams differently fixed, and differently acted upon, are also exactly conformable to that of preceding writers, the errors of which appear in no case to have entered into the contemplation of the author, who, therefore, necessarily arrives at the same conclusions, with regard to the comparative strength of triangular beams in opposite positions, of square beams fixed diagonally and direct, of hollow cylinders, when the bore does not pass immediately through the axis, &c. &c.; the whole of which we have found, both theoretically and practically, to be erroneous. This cannot but be considered as a serious deduction from the general merit of the performance, which, in many respects, displays considerable ingenuity, and probably the most numerous detail of experi-

ments that ever were made by one person. A digression on elastic curves, translated word for word from the memoir of Euler, above referred to, with an investigation of general formulæ for the absolute negative resistance of solids, concludes the first section.

48. The second relates to the investigation of solids of equal resistance, a subject which furnishes much display of analytical transformations, and which may be read with pleasure and instruction by those who are ready in the management and application of fluxional equations; but it must be admitted, that the practical deductions are very few and uninteresting, the principal one being to determine the form and dimensions which the links of a chain ought to have, when they are employed in lifting great weights; so that, considering their own gravity, the whole chain may possess throughout an equal resistance.

49. The third section, which we conceive to be the most useful part of M. Girard's work, is employed in giving the details of his several experiments, a description of the machinery, the various phenomena and irregularities that were observed, &c. &c. These experiments were not made for the purpose of ascertaining the ultimate or greatest resistance of the pieces of timber on which they were performed; the author very properly observing, that this is not the inquiry the most interesting to practical men, as they never want to proportion their timbers so nearly to the load they have to support, as to be very anxious to know the exact

quantity they will carry. It is much more important to be informed as to the degree of deflection a given pressure will produce; for this last, carried beyond a certain point, is often nearly as fatal in the construction in which it occurs as an absolute fracture.

M. Girard's experiments were, therefore, principally directed towards this determination, and were confined to the two woods most commonly employed in building, viz. fir and oak; but it was the latter only which he submitted, experimentally, to a pressure parallel to the direction of its fibres. Here, as might be expected, considerable irregularity was observed, some pieces curving in one direction, and some in another, while others were bent in different directions at the same time, and some pieces were broke with weights which caused but a slight inflection in others of equal or even of less dimensions.

50. All these anomalies must be looked for in experiments of this kind. If the fibres of the wood were completely uniform, both with regard to strength and position, the results would undoubtedly approach towards an equality or uniformity; but when knots, as is generally the case, are found in various parts of the length of a piece of timber, and differently posited in respect of its other dimensions, these will facilitate the bending in those parts where they are found in greatest number; and consequently no very decided conclusion is to be expected.

It is, however, different in experiments on the transverse strain; for the force being there applied to a determined point, the effect will be produced at that point, although perhaps in a greater or a less

degree, or the same effect will be produced with a less or greater force, depending upon the state of the fibres in and about the plane of fracture. Much more uniformity may, therefore, be expected in this case than in the former; and yet even here a discrepance is sometimes observed between the results of two experiments, amounting to a ninth, or even more, of the whole weight employed.

51. The following is an abstract from M. Girard's first and second tables. Table I. contains the results of his experiments on the vertical pressure of oak beams. The 1st column contains the number of the experiments; the 2d, the dimensions and specific gravity* of the several pieces; the 3d and 4th, the height from the bottom to the point of greatest curvature; the former in the direction of the least thickness, and the latter in that of the greatest. The 5th and 6th exhibit the measure of the greatest deflection; the former in the direction of the least, and the latter in that of its greatest dimension: the 7th column shews the several weights under which those deflections were observed; the 8th, the time from the commencement; and the 9th contains remarks, &c.

We have only shewn the effect of four different weights for each beam; but the author himself has, in some cases, employed ten, twelve, or more different pressures, measuring the deflection, &c. for each: but as it was thought unnecessary to follow

* M. Girard gives only the weight of the pieces; but we have preferred changing the weights into the specific gravities, as furnishing a readier means of comparing one piece with another.

him through the whole, the results of his first two and last two charges in the first eighteen experiments have been given. Those columns also, which are drawn from computation founded on the theory of M. Girard, are omitted.

Table II. which is an abstract from the author's second table, contains the results of his experiments on the transverse deflection of such of the beams as were not broken in the experiments above referred to: they were supported at each end at different lengths, and in different positions, viz. first with their greatest thickness vertical, and then with their less.

52. The purpose of these, and various other experiments detailed by this author, was to ascertain what is termed the absolute elasticity of fir and oak; or, which amounts to nearly the same, the relation between the length, depth, breadth, and deflection of different beams, loaded with different weights. It appears from these, that the deflection is directly as the weight and square of the length, and inversely as the breadth and square of the depth. This, at least, is the case with the fir beams: in those of oak, the deflection is as the weight into a certain function of the length, viz. as $P \times \frac{f^3}{f \times \cdot 03}$, (where f is half the length of the beam, and P half the weight with which it is loaded;) and inversely as the breadth and square of the depth, as before.

The two formulæ which the author gives for determining these quantities, the one from the other, are as follow; viz. in,

$$1 \textit{ oak beam} \ldots\ldots \frac{P f^2}{3\, b} = \frac{(11784455)\,(f + \cdot 03)\, a h^2}{1\cdot 3}$$

$$2 \textit{ fir beams} \ldots\ldots \frac{P f^{2*}}{3\, b} = (8161128)\, a h^2$$

P denoting half the weight, f half the length, a the breadth, h the depth, and b the versed sine of deflection: consequently, any four of these being given, the fifth may be determined. All the above dimensions are in metres, and the weight in kilogrammes.†

It is to be observed, that these formulæ belong exclusively to rectangular beams supported at each end, and loaded in the middle; and to connect them with beams of other forms, or of the same forms, but under different circumstances of fixing, &c., the theory and formulæ of Leibnitz are employed; by which means all the errors of the latter are so blended with the former, as to render them wholly useless in all such cases as those to which we have alluded.

We shall have occasion to refer again to the deductions of this author in some subsequent articles, and shall, therefore, pursue our remarks on this performance no farther in this place: it will be sufficient here merely to observe, that the experiments which it details are very numerous; but the author, by aiming at a precision which the nature of the subject will not admit of, and by introducing a calculus

* The formula actually given for fir beams, page 184, and which is employed to the end of the volume, is $\frac{P f^3}{3\, b} = (8161128)\, a h^2$, but it is obviously erroneous.

† It will be seen in the Third Part, in treating of the deflection of beams, that both our theory and experiments give very different results from the above.

much too refined for the matter under consideration, at the same time that he has entirely neglected to investigate the theory of resistances, has certainly not derived from his experiments all the advantages we might otherwise have expected.

53. It may be proper to add here the formulæ M. Girard employs to compute the weight under which a piece of timber ought to begin to bend when pressed vertically, from the deflection being given when charged with any weight horizontally. These are as follow:

Let P represent half the weight when the piece is charged horizontally in the middle, and b the corresponding deflection; f half the length of the piece, and π the semicircumference of a circle to diameter 1.

Then $E\,k\,k = \frac{P f^3}{3\,b} =$ *absolute elasticity.*

And $Q = \frac{\pi^2 E\,k\,k}{4 f^2} = \frac{\pi^2 P f}{12\,b} =$ *the weight* under which the same piece will begin to bend when the pressure is vertical.

If, therefore, for the same depth and thickness, $E\,k\,k$ were constant, the weight Q, under which a piece begins to bend, would be inversely as the square of the length: but M. Girard finds $E\,k\,k$ nearly as the square of the length, or as f^2; and consequently Q varies, *cæteris paribus*, as f inversely.

Note.—In the following table, where two heights and two versed sines are connected by a {, with one weight, it shews that the piece bent in two places, in opposite directions.

TABLE I.

54. GIRARD'S *Experiments on the Vertical Pressure and Resistance of Oak-Beams, or Columns.*

No. of Experiment.	*Dimensions and Specific Gravity of the pieces.*		*Height of the greatest Curvature from the foot.*		*Versed Sine of greatest Curvature.*		*Weight in kilogrammes.*	*Time in hours.*	*REMARKS*
			In the direction of the thickness.	*In the direction of the breadth.*	*In the direction of the thickness.*	*In the direction of the breadth.*			
		metres.	metres.	metres.	metres.	metres.			
1	Length	2·5979	1·0689	1·2989	·0068	—	17320	0·83	Recovered its first form.
	Breadth	0·1580	—	—	·0070	·0068	29691	2·08	
	Thickness	0·1285	—	—	·0090	·0090	37429	2·91	
	Sp. gr.	1038	—	—	·0113	·0090	42418	9·58	
2	Length	2·5979	1·1907	1·2989	·0056	·0045	11993	2·50	Broke under the last weight.
	Breadth	0·1624	0·9742	—	·0068	·0079	25664	9·16	
	Thickness	0·1060	—	—	·0068	·0135	42514	10·83	
	Sp. gr.	984	—	—	·0068				
3	Length	2·5979	0·6495	—	·0023	—	11991	0·83	Recovered its first form.
	Breadth	0·1579	1·9484	—	·0017				
	Thickness	0·1050	1·7861	—	·0113	—	28575	9·58	
	Sp. gr.	1010	1·6237	—	·0282	—	31339	10·41	
4	Length	2·5979	1·2989	1·2989	·0113	·0079	11993	6·66	Did not recover its first form.
	Breadth	0·1330	1·2989	1·2989	·0180	·0124	17341	8·33	
	Thickness	0·0992	1·2989	1·2989	·0497	·0124	22939	10·00	
	Sp. gr.	1000							
5	Length	2·5979	1·2989	1·1366	·0169	·0068	11996	6·66	Broke under the last weight.
	Breadth	0·1308	1·2989	0·9742	·0372	·0113	17341	8·33	
	Thickness	0·1060	—	—	—	—	22931	8·75	
	Sp. gr.	923							
6	Length	2·2731	1·2989	1·1366	·0023	·0045	28619	9·58	Broke under the last weight.
	Breadth	0·1556	1·2989	1·1366	·0023	·0045	33115	16·66	
	Thickness	0·1308	1·4613	1·2989	·0090	·0034	47073	19·58	
	Sp. gr.	920	—	—	—	—	52270	22·50	
7	Length	2·2731	1·6237	0·9742	·0040	·0040	22934	2·08	Recovered its first form.
	Breadth	0·1579	1·6237	0·9742	·0040	·0040	28612	2·91	
	Thickness	0·1285	1·2989	1·2989	·0169	·0045	47047	20·83	
	Sp. gr.	973	1·2989	1·2989	·0186	·0090	47032	23·33	
8	Length	2·2731	1·6237	1·2989	·0062	·0034	17320	12·08	Broke under the last weight.
	Breadth	0·1556	1·6237	1·2989	·0096	·0034	22936	12·91	
	Thickness	0·1038	1·6237	1·2989	·0181	·0045	28616	13·75	
	Sp. gr.	972	—	—	—	—	33120	13·95	
9	Length	2·2731	1·4613	1·2989	·0068	·0068	17321	12·08	Broke under the last weight.
	Breadth	0·1579	1·2989	1·1366	·0124	·0045	22940	12·58	
	Thickness	0·1015	—	—	—	—	26626	14·50	
	Sp. gr.	926							

TABLE I.

Girard's *Experiments on the Vertical Pressure and Resistance of Oak-Beams, or Columns.*

No. of Experiment.	Dimensions and Specific Gravity of the pieces.		Height of the greatest Curvature from the foot. In the direction of the thickness.	Height of the greatest Curvature from the foot. In the direction of the breadth.	Versed sine of greatest Curvature. In the direction of the thickness.	Versed sine of greatest Curvature. In the direction of the breadth.	Weight in kilogrammes.	Time in hours from commencement.	REMARKS.
		metres.	metres.	metres.	metres.	metres.			
10	Length	2·2631	1·4613	1·1366	·0079	·0062	11999	10·00	Recovered its first form.
	Breadth	0·1262	1·4613	0·9742	·0079	·0062	15025	12·91	
	Thickness	0·1015	1·4613	0·9742	·0113	·0062	17320	22·91	
	Sp. gr.	1038	1·2989	0·9742	·0135	·0068	20326	25·83	
11	Length	1·9484	0·9742	0·9742	·0045	·0079	17321	7·08	Recovered its first form.
	Breadth	0·1556	0·9742	0·9742	·0045	·0090	22940	10·00	
	Thickness	0·1330	0·9742	0·9742	·0051	·0101	33105	26·66	
	Sp. gr.	1102	—	—	—	—	39644	27·50	
12	Length	1·9484	0·9742	0·9742	·0079	—	22940	20·00	
	Breadth	0·1579	0·9742	0·9742	·0079	—	33123	25·00	
	Thickness	0·1308	0·6495	0·9742	·0068	·0146	39637	27·91	
	Sp. gr.	935	1·6237		·0023				
13	Length	1·9484	0·6495	0·6495	·0045	·0056	17321	2·08	Recovered its first form.
	Breadth	0·1579	0·6495	0·6495	·0062	·0068	22939	3·33	
	Thickness	0·1015	0·6495	0·6495	·0068	·0108	39456	33·33	
	Sp. gr.	987			·0023				
14	Length	1·9484	1·4613	0·9742	·0045	·0040	11973	10·00	Broke under the last weight.
	Breadth	0·1601	1·4613	0·9742	·0045	·0045	17274	27·50	
	Thickness	0·1015	1·6237	1·2989	·0113	·0090	28509	40·41	
	Sp. gr.	1035	—	—	—	—	32996	50·41	
15	Length	1·9484	1·2989	0·6495	·0056	·0045	17294	10·00	Recovered its first form.
	Breadth	0·1330	0·9742	0·6495	·0051	·0045	22899	28·33	
	Thickness	0·1060	1·6237	1·4342	·0068	·0118	46952	86·66	
	Sp. gr.	1032	0·3247		·0011				
16	Length	1·9484	0·9742	0·9742	·0045	·0056	11998	18·33	Broke under the last weight.
	Breadth	0·1285	0·9742	0·9742	·0056	·0079	17317	20·00	
	Thickness	0·1082	0·6495	0·6742	·0045	·0135	37273	92·50	
	Sp. gr.	993	0·2435		·0011				
17	Length	2·2731	0·6495	0·9742	·0029	·0028	11998	10·00	Recovered its first form.
	Breadth	0·1579	—	0·6495	—	·0034	17320	20·00	
	Thickness	0·1082	1·6237	0·9742	·0056	·0045	33120	52·50	
	Sp. gr.	920	1·4613	0·9742	·0113	·0051	39630	57·50	
18	Length	2·5979	0·9742	1·2989	·0051	·0034	11999	10·00	Broke under the last weight.
	Breadth	0·1579	0·9742	1·2989	·0068	·0056	17321	20·00	
	Thickness	0·1353							
	Sp. gr.	916	1·6237	1·2989	·0146	·0079	37305	50·83	

TABLE II.

55. Girard's *Experiments on the Deflection of Oak-beams, when supported at the Ends, and loaded in the Middle of their Length.*

Note.—These Beams are the same as those in the preceding Table.

No. of Experiment.	Dimensions, in metres, and Specific Gravity of the pieces.		Deflection in metres.	Weights in kilogrammes.	Absolute elasticity, computed from $\frac{P f^3}{3 b}$.*	REMARKS.
1	Length	2·5978	·0180	1884	38250	These two experiments were performed on the same piece of wood, which was also the same as No. 1. first Table.
	Depth	·1579	·0238	2379	36524	
	Breadth	·1285	·0238	2932	37876	
	Sp. gr.	1038	·0373	3467	33954	
2	Length	2·2731	·0158	1884	29174	
	Depth	·1579	·0215	2395	27266	
	Breadth	·1285	·0248	2930	28909	
	Sp. gr.	1038	·0300	3470	28304	
3	Length	1·9484	·0045	1882	64461	These were all the same piece of wood, viz. No. 7. of the first Table.
	Depth	·1579	·0056	2393	65864	
	Breadth	·1285	·0113	4007	54634	
	Sp. gr.	973	·0153	4542	45744	
4	Length	1·6237	·0056	1877	29893	
	Depth	·1579	·0068	2388	31312	
	Breadth	·1285	·0119	4976	37288	
	Sp. gr.	973	·0141	5512	34844	
5	Length	1·6237	·0056	1876	29877	
	Depth	·1285	·0079	2388	26953	
	Breadth	·1579	·0119	3463	34313	
	Sp. gr.	973	·0158	4000	29973	
6	Length	1·9484	·0090	1874	32101	This piece was No. 13. of Table I.
	Depth	·1579	·0135	2385	27228	
	Breadth	·1015	·0271	3786	22669	
	Sp. gr.	987	·0316	4519	22038	
7	Length	1·9484	·0135	1874	21395	
	Depth	·1015	·0226	2383	16313	
	Breadth	·1579	·0271	2919	16500	
	Sp. gr.	987	·0474	3448	11212	
8	Length	1·9484	·0158	1871	18257	This piece was No. 15. of Table I.
	Depth	·1330	·0180	2380	20381	
	Breadth	·1060	·0282	2917	15942	
	Sp. gr.	1032				
9	Length	1·9484	·0232	1870	12437	
	Depth	·1060	·0893	2916	13267	
	Breadth	·1330				
	Sp. gr.	1032				

* P = half the charge, f half the length, and b the deflection.

The 3d and 4th tables of M. Girard contain the detail of similar experiments to those given above in Table II.; except that they were made on what the French workmen call *bois du brin*, which appears to be analogous to our term *balks;* viz. pieces the whole size of the tree, except what is cut off to render them square: each piece was charged at four, or more, different lengths, and in different positions; viz. sometimes with the greatest dimension vertical, and sometimes the least; and the deflections were ascertained for ten, twelve, or more, different weights, and the corresponding elasticity computed for each.

We shall here conclude this historical sketch, reserving any farther observations on the results of the preceding experiments, till we have to compare them with our own, or with each other, in our subsequent theoretical investigations.*

* Since the preceding pages were written and arranged for the press, I have had an opportunity of consulting a memoir of M. Dupin, corresponding member of the National Institute, published in the 10th Vol. of the Journal de l'Ecole Polytechnique. This author has made many very accurate experiments on the deflection of wood, as depending upon its length, breadth, depth, &c.; and as most of the experimental and theoretical deductions in the following pages, relative to this part of the subject, are at variance with those of M. Girard, it gave me great pleasure to find so accurate an agreement between the results of this gentleman and my own. I have since had the honour of several personal interviews with M. Dupin, who was present while I performed some experiments, and who has himself continued to pursue the subject with great attention and ability. Some particulars relative to M. Dupin's results will be found in Part III.

AN ESSAY

ON THE

STRENGTH AND STRESS OF TIMBER.

PART II.

CONTAINING THE DETAIL OF EXPERIMENTS, &c.

An Explanation of the Method of making the Experiments on the direct Cohesion of different Woods, Description of Apparatus, &c.

56. It has been before remarked, that notwithstanding the mechanical operation in this kind of fracture is by far the most simple of the four to which we have alluded, yet it is the most difficult to submit to actual experiment; and it was not till after some consideration, and one or two failures, that we were led to adopt the apparatus exhibited in Plate IV.

Here A B, Fig. 1, represents one of the pieces whose strength is to be determined, its whole length being 12 inches; the length of each square end $3\frac{1}{2}$ inches, and the side of the square end $1\frac{1}{2}$ inch; the intermediate part of 5 inches was turned in an excellent instrument, by a very correct workman,* and brought down in the centre to $\frac{1}{3}$d or $\frac{1}{4}$th of an inch

* Mr. Short, modeller to the Royal Military Academy.

in diameter;* but the other cylindrical parts were made each $\frac{3}{4}$ inch in diameter. CC, DD, fig. 2, represent two strong iron bars, brought to the form shewn in the plate; GG are two screws which are passed through the holes HH, in the bar DD, and are there screwed fast by the nuts I, I; E, E, are two semicircular collars, rivetted one to each bar, which, when the two are fixed together, form a circular plate, as represented in fig. 4. The circular hollow parts *e*, *e*, are $\frac{3}{4}$ inch in diameter, so as to fit exactly the larger part of the cylinder shewn in fig. 1. These bars, after being screwed together, were rested on their supports, as in fig. 4, and, as the workmen express it, *brought out of winding*, and accurately adjusted to a horizontal position by a spirit level.

The two iron boxes MNO, $M'N'O'$, fig. 3, were made exactly to fit the square head B of fig. 1, having also two semicircular holes at top, correctly fitted to the larger part of the cylinder: these were shut by passing the bolts N', M', through the holes N, M, and were thus secured by the two shears shewn in fig. 4.

Having thus described the separate parts of the apparatus, the reader will perceive at once the manner in which they were employed in the experiment; viz. the head A of fig. 1, was placed above the collar

* As it was difficult to measure very exactly the diameter of the small cylinder, it was found by winding a fine thread of silk ten times about it, and then dividing its length by the number of volutions, in order to get the mean circumference, and hence the diameter.

EE, fig. 2, the upper larger cylindrical part of fig. 1 being placed in the hollow parts *e, e*, of fig. 2, when the two parts were securely fixed together by the nuts and screws, I, G; I, G. In the same manner the lower end B, of fig. 1, was enclosed in the two iron boxes MNO, $M'N'O'$, fig. 3, and fastened in that position by means of the bolts $M'N'$, and the shears above described. The whole was then rested on the props fig. 4; and the hook of the scale being inserted in the circular hole formed by O, O', fig. 3, the whole was ready for the experiment, as shewn at large in the former figure.

Every thing being thus prepared, the wedges shewn in the plate were introduced under the scale, to keep it steady, while the larger weights were put in; the former were then removed, and smaller weights added in succession till the fracture took place.

The weights were 10 inch, 8 inch, and $5\frac{1}{2}$ inch shells, loaded each with as many musket balls, as brought them respectively to 100lbs., 50lbs., and 15lbs. A few common weights of 7lbs., 4lbs., 2lbs. &c. were also employed toward the conclusion of an experiment, where it was necessary to increase the weight by small degrees.

It should also be observed, that as a slight vibration of the scale might cause a fracture in the small cylinder, submitted to the operation of the weight, four small braces were made use of, one at each corner of the scale, to prevent any such motion. These were attached to the four inward legs of the stand, which are omitted in the plate, to avoid a complica-

tion of parts: they were the same as shewn in Plate V., and were fixed down to the floor in the same manner as described in Article 62.

57. The results of these experiments are exhibited in the following Table.

TABLE I.

58. *Experiments on the Direct Cohesion of different Woods.*

No. of Experiments.	Names of the Woods.	Specific Gravity.	Circumference.	Weight in lbs.	Weight reduced to a square inch.	Mean value of direct cohesion on a square inch.
1	Fir	600	1·05	1140	12993	12857
2	do.	600	1·10	1260	13073	
3	do.	600	1·10	1191	12037	
4	do.	600	1·05	1160	13220	
5	do.	600	1·11	1213	12371	
6	do.	600	1·05	1180	13448	
7	do.	581	1·10	1059	11000	11549
8	do.	564	1·10	1201	12472	
9	do.	601	1·10	1094	11360	
10	do.	611	1·10	1130	11736	
11	do.	532	1·10	1076	11180	
12	do.	590	1·10	1112	11548	

The first six experiments were made upon the fragments of the four foot pieces (Art. 71), which were the same also as the triangular pieces, Nos. 3, 4, 7, and 8 (Art. 76), were cut from.

These pieces were all cut from a plank remarkably free from knots and irregularities, which throughout gave more uniform results than any other specimen; on which account the several results derived from them have been adopted in a subsequent section, to investigate the position of the neutral axis, and the centres of tension and compression.

No. 7, broke by a part of the fibres drawing out of the head of the piece: it was probably first broken by an accidental motion of the scale.

No. 9, broke by the whole of the middle cylinder drawing out of the head, to the length of about 2 inches, where there was a knot, which might break off the continuation of the fibres. The others were all complete fractures.

TABLE I.

59. *Experiments on the Direct Cohesion of different Woods.*

No. of Experiments.	*Names of the Woods.*	*Specific Gravity.*	*Circumference.*	*Weight in lbs.*	*Weight reduced to a square inch.*	*Mean value of direct cohesion.*
13	Ash	594	·8800	1100	17850	
14	do.	611	·9000	1096	17003	17207
15	do.	611	·8750	1024	16770	
16	do.	600	·8375	881	15784	
17	do.	600	·8625	1025	17315	16947
18	do.	600	·8750	1081	17742	
19	Beech	712	·880	716	11626	
20	do.	694	·890	721	11437	11467
21	do.	700	·900	731	11338	
22	Oak	770	1·10	856	8889	
23	do.	770	1·10	887	9211	9198
24	do.	770	1·10	908	9494	
25	do.	920	·8800	740	12008	
26	do.	920	·8750	712	11660	11580
27	do.	920	·8900	698	11072	

Nothing remarkable happened in the course of these experiments, except that No. 4 of the ash, viz. No. 16 above, was observed to twist, during the action of the weight, about 7°½, but the fracture took place in the small part of the cylinder: as this piece, however, bore less weight than any other of the ash, it is probably to be attributed to the above circumstance; a similar effect was observed in our specimens of mahogany, as stated in the following page.

It is proper to observe, that Nos. 13, 14, and 15 were made from the fragments of the 2 inch square ash pieces, Table XI.; those of the beech from the fragments of the similar pieces, Table XII.

The three first oak pieces were off the same plank as the several battens, Table IX. It was a very fine piece of English oak, which had been a considerable time in store, and was perfectly dry: the other specimen, viz. Nos. 25, 26, and 27, appears, from its specific gravity, to have been more recently felled: it was also of a closer texture.

TABLE I.

60. *Experiments on the Direct Cohesion of different Woods.*

No. of Experiments.	*Names of the Woods.*	*Specific Gravity.*	*Circumference.*	*Weight in lbs.*	*Weight reduced to a square inch.*	*Mean value of direct cohesion.*
28	Teak	860	·8625	868	14662	
29	do.	860	·8625	900	15203	15090
30	do.	860	·8625	912	15405	
31	Box	960	·8625	1168	19730	
32	do.	960	·8625	1160	19595	19891
33	do.	1024	·8625	1200	20348	
34	Pear	646	·8625	683	11537	
35	do.	646	·8500	523	9096	9822
36	do.	646	·8625	523	8834	
37	Mahogany.	637	1·1125	783	7950	
38	do.	637	1·1125	783	7950	8041
39	do.	637	1·1125	810	8224	

Nos. 28, 29, and 30 were from a piece of teak, which had been taken from an old ship. Some other specimens were tried, but the results were so irregular, that it would be useless to give them; and exactly the same occurred in the first experiments on the transverse strain of this wood.

In the two first experiments on box, the small part of the cylinder drew out of the head, which was 5¼ inches in length, but not so perfectly as in the fir piece already mentioned; the part that drew out being very tapering, so that we could but barely see through the hole thus formed. It is therefore obvious that, although the mean strength amounts to nearly 20,000 lbs. upon a square inch, this is still short of the absolute strength of direct cohesion of this wood.

The same may be observed with regard to the mahogany, but it proceeded from a different cause; viz. the twisting of the pieces, which, in all the experiments, wrenched the fibres asunder, instead of drawing them apart. The effect seems to have been exactly the same as would happen to a weight suspended to a rope, which would have a tendency to untwist; and it is highly probable that the fibres of the tree had acquired, in their growth, a similar situation with regard to each other as the component fibres of the rope, but of course in a much smaller degree.

TABLE II.

61. *Experiments on the Lateral Adhesion of Fir.*

No. of Experiments.	*Names of the Woods.*	*Length drawn out.*	*Circumference.*	*Weight in lbs.*	*Weight reduced to one inch surface.*	*Mean value of lateral cohesion on one inch surface.*
1	Fir.	1·625	1·1	996	556	592
2	do.	1·625	1·15	1187	621	
3	do.	1·625	1·15	1117	584	
4	do.	1·375	1·15	1066	634	
5	do.	1·500	1·15	1000	578	
6	do.	1·500	1·15	1000	578	

It is stated in a few of the preceding experiments, that the fibres, instead of breaking, as was intended, in some instances drew out, either wholly or in part, from the head of the pieces, notwithstanding these were, in one instance, more than 5 inches in length. This circumstance suggested the above experiments, in which the head of the piece was bored down very accurately to the distances stated above in the third column, viz. to the insertion of the smaller cylinder into the greater part, see *fig.* 1. *plate* IV.; the several pieces were then suspended, as in the foregoing experiments, and the weights put on as usual, till the separation took place; that is, till the small part was drawn out, or broken.

Nos. 1, 3, and 5 were drawn out very completely; the part which came out being nearly as perfect a cylinder as that which was turned: the other three were more or less irregular.

Nos. 2 and 4 twisted at least 10° before the separation took place.

It appears from the above, that the lateral adhesion is not more than one-twentieth of the direct cohesion in fir; with the other woods we did not attempt any experiments.

62. From a mean derived from the preceding experiments, and employing only the nearest round numbers, it appears that the strength of direct cohesion on a square inch of

	lbs.
Box, is about	20,000
Ash	17,000
Teak	15,000
Fir	12,000
Beech	11,500
Oak	10,000
Pear	9,800
Mahogany	8,000

Also, that the strength of the lateral adhesion of the fibres in fir is about equal to 600lbs. on a square inch.

Some of these numbers differ considerably from those given by Musschenbroeck, as is stated in Art. 4.; on which head it will be sufficient to observe, that the preceding experiments, from which the above results are drawn, were made with every possible care that the delicacy of the operation required.

63. *Practical Rule.*—Since the strength of direct cohesion must necessarily be proportional to the number of fibres, or to the area of the section, it follows, that the strength of any rod will be found by multiplying the number of square inches in its section by the corresponding tabular number, as given above.

This, however, gives the absolute and ultimate strength of the fibres; and therefore, if the quantity that may be safely borne be required, not more than

two-thirds of the above values must be used, or perhaps not more than one-half; although in several of the foregoing experiments, more than ⅘ths of the whole weight have been left hanging for 24 or 48 hours without our perceiving the least change in the state of the fibres, or any diminution of their ultimate strength.

An Explanation of the Method of making the Experiments on the Transverse Stress and Strain of Battens of different Woods, with a Description of the Apparatus, &c.

64. These experiments may be divided into four classes, viz. 1st, when the battens were supported on two props, as shewn in Plate V. 2dly, When they were fixed horizontally, with one end in a wall, as in fig. 3, Plate VI. 3dly, When they were fixed at any given angle, as shewn in fig. 1 and 2, Plate VI.; and, lastly, When both ends were firmly fixed, as in fig. 4 of the same plate.

Plate V. represents an experiment on a fir batten, AB, 7 feet in length, and two inches square, resting on the two props CD, EF, 6 feet asunder: the two weights PP are 1 lb. each, and were used to keep the fine silk line, to which they were attached, stretched in a horizontal position between the props: to facilitate which, the line was made to pass over two small brass rollers, one of which is shewn at G. By means of this line, and the several small scales, *s*, *s*, *s*, &c., each divided into 10ths of inches, the deflection of the batten might be observed with great accuracy; and in

this manner those given in the detail of the experiments were taken.

The number of these scales were varied at pleasure: commonly there was only one in the centre; at other times we had from 3 to 10, or even more; and in some few cases a board was placed against the batten, and the curve traced upon it with a pencil.

The small ivory scale at H was intended to measure the successive lengthening or stretching of the lower fibres, and was thus adjusted:—

A fine silk line was fixed at the end A of the batten AB, and brought under the whole length of A to B: the scale, which had two fine steel points attached to it, was fixed by them into the under side of the batten, as shewn at H: at the top of the scale was a small brass wheel or roller, over which the silk passed, and to the end of which was hung a small semicylindrical brass weight, with its flat side towards the scale: two fine grooves were also cut, one in each of the brass plates, with which the tops of the props CD, EF, were defended, in order to allow the silk line to pass freely in them under the piece.

The batten thus furnished was now rested on the two props, with the line placed so as to pass in the two grooves above mentioned; and by means of a screw, by which the line was attached to the piece at A, the weight at H was adjusted to 0, on the same scale, which was divided from 0 upwards into 40ths of inches.

It is obvious now, that after the weights began to give the batten any deflection, the small weight at H will be raised along the scale by a quantity

exactly equal to the difference between the original length of the bottom fibres, and the length to which they are stretched at the time of making the observation; and in this manner the stretching of the fibres at several different degrees of deflection was measured in a few experiments: but as it did not appear that any useful application of this datum could be made in the theory, and as it required a longer time to adjust, &c., it was employed, comparatively, but in a few cases.

It would be useless to enter more minutely into an explanation of these experiments, as the process will be obvious from an inspection of the plate: we shall therefore merely observe, that the artist has chosen to represent the apparatus as if the experiments were performed in the open air; and the consequence is, that the props do not appear sufficiently steady: it is therefore proper to inform the reader, that the experiments were performed under cover, on a substantial floor; and the trestles or props were made to slide in grooves, and were firmly fixed in them, so as to render the whole perfectly secure and steady: and, to prevent any momentum in loading the scale, this was always made stationary by wedges, when the larger weights were introduced.

65. In order to make the experiments on those pieces which were fixed by one end in a wall, the following means were employed. A block of hard wood, A B C D, fig. 3, Plate VI. about 18 inches long, and 12 inches in breadth and depth, was cut through at about 5 inches from each end, as at *ab cd*, for the convenience of forming a hole

2 inches in breadth and depth, or rather more; the one with the side of the square vertical, and the other with the diagonal vertical, as shewn in the figure. The parts of the block were then screw-bolted together; and an iron socket, exactly two inches square on the inside, was made to fit these holes very accurately, but so that it might be taken out and put in at pleasure: a hole was then cut out of a very heavy solid wall, a little larger than the block, and into which the latter was fixed by means of inverted wedges, whereby the whole was rendered perfectly firm and immovable.

The pieces of timber on which the experiments were made were two inches square, and therefore fitted tight into the iron sockets above mentioned, the edges of which are shewn in the figure; the under side being made slightly curving, to prevent the cutting of the lower face of the piece after the weight was hung on: and as the deflection would have rendered the scale liable to slip off, an iron plate, with two studs rivetted to it, was screwed on the end of the batten, as shewn at E and F, the former being bent into a right angle to fit its upper edge.

In the same manner the blocks of fig. 1 and 2 were made and fixed, differing from the former in nothing except the hole being made to form an angle of 26° with the horizon; the first ascending, and the other descending.

Those of figure 4 were precisely the same as the lower part of fig. 3, and were fixed into two walls exactly 6 feet asunder.

Every thing being thus adjusted, the scale was hung on, as shewn in Plate V., but which, for

simplicity, is merely represented, in Plate VI., by a single ball, W.

66. It may not be amiss to add, that the walls in which the blocks were fixed were not less than 40 feet high, although in the plate they are represented as if they were not above 6 feet; it being thought useless to shew them in their full height.

Such were the means employed for assuring accuracy in the results, and which it has been thought right to explain at length, in order that the reader may judge of the degree of confidence to which these experiments are entitled. This has been commonly omitted by preceding authors, and has been the subject of just complaint by those who would have wished to avail themselves of their results for the purpose of theoretical investigation; so that in cases where a disagreement was found to have place between the theoretical and practical results, it was always doubtful to which the error belonged, and was therefore attributed to either, as best suited the views of the writer.

The following are the results of the different experiments made on the transverse strain, arranged according to their dimensions.

TABLE III.

EXPERIMENTS

(67.) *On Fir Battens, supported at each End.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Weight in lbs.	Weight reduced to Sp. Gr. 600.	Mean Weight corresponding to Sp. Gr. 600.
1	15	1	1	504	360	428	439
2				533	388	436	
3				564	418	444	
4				646	453	421	
5				588	453	462	
6				600	441	441	
7	18	1	1	552	318	346	342
8				647	364	338	
9				724	436	371	
10				719	404	337	
11				648	353	327	
12				672	376	336	

The above experiments were made principally in order to determine what relation there might be between the ultimate strength and the specific gravity of the rods: they were therefore selected out of those which had been the same time in store, and that differed the most from each other in their gravity, and principally from the fragments of those that had been broken in preceding experiments, of which the detail is given in the subsequent pages.

The reduced weight in the seventh column above is found on a supposition that the strength is as the specific gravity: a reduction which is adopted throughout.

We can see no physical reason for the circumstance of the strength being so nearly proportional to the specific gravity. It ought rather, one would have supposed, to have been as the $\frac{2}{3}$d power; for, supposing the number of particles to be as the specific gravity, the number of them in any section would be as the $\frac{2}{3}$d power of the latter. Upon the whole, however, the simple ratio of the strengths being as the specific gravities, seems to answer better than any other.

TABLE III.

EXPERIMENTS

(68.) *On Fir Battens, supported at each End.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Deflection.	Specific Gravity.	Weight in lbs.	Weight reduced to Sp. Gr. 600.	Mean Weight, Sp. Gr. 600.
1	24	1	1	1·25	—	270	—	265
2				1·25	—	262	—	
3				1·25	—	262	—	
4				—	560	261	279	288
5				—	560	283	303	
6				—	540	256	284	
7	30	1	1	1·80	—	242	—	237
8				1·80	—	234	—	
9				1·80	—	235	—	
10	36	1	1	1·85	577	229	237	196
11				3·12	505	162	192	
12				3·00	505	148	160	
13				2·2	553	181	196	
14				3·2	553	181	196	
15				2·2	553	181	196	

The specific gravities of Nos. 1, 2, and 3 were not observed, nor the deflections of 3, 4, and 5. The deflections of 1, 2, and 3 were all the same, viz. for 220 lbs. $\frac{11}{16}$ inch; for 250 lbs. one inch; for 260 lbs. $1\frac{1}{4}$ inch.

The specific gravities of Nos. 7, 8, and 9 not observed; these, with Nos. 1, 2, and 3, were broken, before it was thought necessary to introduce that consideration.

Nos. 11 and 12 were both off a very light plank, and were very elastic.

Nos. 13, 14, and 15 were very uniform rods: Nos. 13 and 15 were bound to two pieces of the same thickness as themselves, but each half the whole length, to prevent any curving; and No. 14 was broken as usual. It seems, therefore, that the curving of the batten does not weaken it, although it increases the deflection.

TABLE III.

EXPERIMENTS

(69.) *On Fir Battens, supported at each End.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Deflection.	Specific Gravity.	Weight in lbs.	Reduced to Sp. Gr. 600.	Mean Weight. Sp. Gr. 600.
1	24	1½	¾	—	646	420	390	397
2				—	646	424	393	
3				—	646	441	409	
4				·70	746	557	448	435
5				·70	709	501	424	
6				·70	734	531	434	
7	30	1½	¾	1·12	733	412	337	336
8				1·12	733	411	336	
9				—	646	360	334	

No. 1 was a very complete fracture, shewing very distinctly the part of the section which had been compressed, and that which had acted by tension; the latter rather exceeded $\frac{1}{3}$ of the whole depth. In Nos. 2 and 3 the same appearance might be observed, but not so perfectly. No. 3 hung two hours and a half before breaking; the others only ten minutes.

Nos. 4, 5, and 6 were remarkably sound pitch pine, full of turpentine. No. 5 would probably have borne as much as No. 4 or No. 6, but that the upper part, on which the weight hung, was more tender, and was much crippled in the experiment.

Nos. 7 and 8 were part of the same plank as Nos. 4, 5, and 6; and No. 9 was part of the specimen from which Nos. 1, 2, and 3 were made.

It appears from the first of the above set of experiments, that the strength is in a higher ratio than that of the specific gravities.

TABLE III.

EXPERIMENTS

(70.) *On Fir Battens, supported at each End.*

No. of Experiments.	*Length in inches.*	*Depth in inches.*	*Breadth in inches.*	*Deflection.*	*Specific Gravity.*	*Weight in lbs.*	*Reduced to Sp. Gr. 600.*	*Mean Weight. Sp. Gr. 600.*
1				·625	613	1190	1164	
2	24	2	1	—	563	1000	1066	1119
3				—	600	1128	1128	
4				—	586	882	903	
5	30	2	1	—	581	871	901	900
6				1·08	571	852	895	
7				1·00	—	600	—	
8				1·12	—	622	—	
9	36	2	1	1·12	—	680	—	600
10				1·12	—	595	—	
11				1·52	—	552	—	
12				1·50	—	550	—	
13				1·12	606	722	715	
14	36	2	1	1·12	606	752	744	745
15				1·12	564	730	776	

No. 1 was left for twenty-four hours, with 865lbs. hanging upon it, without any deflection beyond what it had acquired in a few minutes.

The successive deflections of No. 6 were 520lbs. = $\frac{5}{16}$ inch, 620lbs. = $\frac{6}{16}$, 720lbs. = $\frac{17}{16}$.

Nos. 7, 8, and 12, were broken before it was thought necessary to introduce the specific gravities; they were lighter and weaker wood than the preceding; and Nos. 5 and 6 were obviously damaged, by being exposed to wet.

The successive deflections and stretching of Nos. 13 and 14 were as follow; viz.

220lbs.	deflection	$\frac{2}{8}$	stretching	0
420	....	$\frac{4}{8}$	......	$\frac{1}{20}$
520	....	$\frac{5}{8}$	......	$\frac{3}{40}$
580	....	$\frac{6}{8}$	......	$\frac{1}{10}$

TABLE III.

EXPERIMENTS

(71.) *On Fir Battens, supported at each End.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Weight in lbs.	Successive Deflections.				Mean Weight reduced to Sp. Gr. 600.
1	44	2	2	630	421	·175	·266	·300		1255
					848	·350	·566	·660		
					1054	·450	·700	·900		
					1166	·530	·900	1·025		
					1211	·600	1·00	1·15		
					1226	·650	1·10	1·30		
					1288	·900	1·57	1·95		
					1317	—	—	2·35		
2	44	2	2	—	421	·175	·275	·350		
					848	·366	·633	·763		
					1054	—	—	2·00		
3	48	2	2	601	421	·15	·25	·33	·36	1116
					711	·27	·47	·60	·66	
					920	·40	·60	·90	1·02	
					1020	·53	·90	1·23	1·4	
					1125	—	—	—	2·3	
4	48	2	2	601	1110	The same deflection.				

The deflections in the above experiments were measured by scales fixed on the pieces at equal distances, from one end to the middle, as explained in Art. 64.

It was remarked, in the experiment No. 1, that the deflection of the piece was very sensibly affected, after 1240lbs. were on, by the addition and subtraction of a 7lb. weight.

No. 2 was part of the same plank as No. 1, and only parted from it by the saw, although it was so much weaker; it was sappy and light, but the account of its specific gravity was lost, or not taken.

In Nos. 3 and 4 seven scales were used, placed at equal distances, viz. one at every six inches. The deflections are only given above from the middle to one end.

TABLE III.

EXPERIMENTS

(72.) *On Fir Battens, supported at each End.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Weight in lbs.	Successive Deflections and Lengthening. Deflections.	Length.	Mean Weight reduced to Sp. Gr. 600.
1	60		2	—	788			
					421	·33 ·56 ·75	·087	
					521	·40 ·70 ·96	·125	
2	60	2	2	—	711	·73 1·30 1·80	·162	770
					811	·93 1·70 2·37		
3	60	2	2	—	711	not observed.		
					221	·35 ·60 ·75	·062	
					421	·70 1·2 1·45	·125	
4	72	2	2	563	521	·90 1·55 1·87	·150	
					621	1·30 2·30 2·80	·187	
					682	— — 4·30	·200	744
					221	·30 ·53 ·65	·075	
					421	·60 1·03 1·20	·162	
5	72	2	2	600	521	·76 1·33 1·50	·187	
					621	1·00 1·70 2·00	·225	
					760	— — 3·50	·350	

But little dependence can be placed upon the experiments Nos. 1, 2, and 3. No. 1 was part of a weak plank; and No. 2 and 3 were cut from one piece, which was at first 8 feet 6 inches; after breaking it at 5 feet, the remnant, which was then 6 feet, was broke again at 5 feet, breaking with the weight stated in No. 3: the latter part was nearest the root end. The specific gravities were not taken.

Nothing particular was noticed in experiments 4 and 5. The lengthening of the piece was measured by means of the instrument described Art. 64. And, in order to protect the battens against the splintering which commonly happened in the preceding experiments, they were bound round with twine on each side of the place of fracture; leaving about two inches clear in the middle.

Observations relative to the preceding Experiments.

73. It is proper here to observe, that the preceding results must not be considered as furnishing any data that are applicable to fir in general; for as the object was principally to ascertain the law which takes place between the strength and the dimensions of the pieces, the greatest care was taken in selecting the best and most perfect specimens of the kind that could be procured: several of the planks had been in store for a considerable time, and were perfectly seasoned, which accounts for the specific gravities being less than is usually found for Riga fir and Christiana deals, of which the specimens principally consisted. By this means a greater uniformity was found in the results, and a greater strength than is generally due to this kind of wood; but the results were obviously so much the better adapted for eliciting a correct theory. The medium strength of Riga fir will be found in the general table of data.

TABLE IV.

MISCELLANEOUS EXPERIMENTS

(74.) ***On Fir Beams, cross cut in the Centre, and supported at each End.***

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Weight in lbs.	Deflection.	Mean Weight reduced to Sp. Gr. 600.
1	30	2	1	581	808	1·00	
					220	·250	
					420	·440	
2	30	2	1	581	520	·500	856
					620	·625	
					780	·750	
					846	·875	
3	30	2	1	580	835	·875	Same deflections as No. 2.

The preceding experiments having shown pretty clearly the situation of the neutral axis; viz. that it was at about $\frac{3}{8}$ths or $\frac{3}{9}$ths of the depth of the section from the bottom; these bars, which were part of the same specimens as those of the same dimensions (Art. 70), were cut down $1\frac{1}{4}$th inch, or $\frac{5}{8}$ths of the depth, and the saw-groove filled up by a thin slip of pear-tree, sufficiently tight to preserve the stiffness of the battens, but without straining them. They were then loaded as usual, and were broken with the weights above stated.

On examining the wedges, or slips of pear tree, after the experiments, it was found that No. 1 was a little longer than No. 3, and No. 3 than No. 2; and the wedge of another batten, that broke with a considerable less weight, was $\frac{1}{16}$th of an inch longer than any of them. The impression of the fibres was very distinctly marked on the wedges; strongest at top, and gradually weakening towards the bottom, where they could scarcely be distinguished.

These experiments seemed to indicate that the neutral axis was very nearly at $\frac{5}{8}$ths of the depth of the batten. The deflection of No. 1 exceeded that of No. 2 and 3 by $\frac{1}{8}$th throughout.

TABLE V.

MISCELLANEOUS EXPERIMENTS

(75.) *On Fir Battens, grooved out in the Centre, and supported at each End.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Weight in lbs.	Deflection.	REMARKS.
1	36	2	1½	564	421 711 1095	·25 ·43 1·0	Whole beam.
2	36	2	1½	564	421 711 985	·300 ·566 1·10	Groove downwards ⅔d inch deep, and ½ inch broad.
3	36	2	1½	538	421 621 780	·366 ·630 1·50	Groove upwards ⅓d inch deep, and ½ inch broad.

These weights, reduced to specific gravity 600, gave No. 1, 1164; No. 2, 1047; No. 3, 870.

The experiments in the preceding page having nearly pointed out the position of the neutral axis, these experiments were made with a view of ascertaining what part of the resistance was due to compression, and what to tension. Nos. 2 and 3 were therefore grooved out, in the centre of their breadth, from end to end; the former to ⅓d of the depth, and the latter to ⅔ds, and each ½ an inch broad; viz. ⅓d of the breadth. The idea was, that what No. 2 broke short of the weight required in the whole batten, would be the measure of ⅓d of the tension; and what No. 3 broke short of the same, would be the measure of ⅓d of the compression. This view of the subject was afterwards found to be erroneous; but the experiments were retained, on a supposition that they might still form some standard of comparison.

TABLE VI.

MISCELLANEOUS EXPERIMENTS

(76.) *On Triangular Fir Battens.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Weight in lbs.	Position of the Battens.	Mean Weight reduced to Sp. Gr. 600.
1	24	$\frac{1}{2}\sqrt{2}$	$\sqrt{2}$	—	118	Base upwards.	
2				—	97	Do. downwards.	
3	24	$\sqrt{3}$	2	613	740	Base upwards.	740
4				588	740	Do. do.	
5				559	680	Do. do.	720
6				574	680	Do. do.	
7	24	$\sqrt{3}$	2	619	637	Base downwards.	626
8				603	637	Do. do.	
9	20	$\sqrt{3}$	2	—	907	Base upwards.	
10				630	843	Do. downwards.	

These pieces were made out of the fragments of the 2-inch square battens; viz.

3 and 8 out of No. 3, art. 71.
4 and 7 out of No. 4, art. 71.
5 and 6 out of No. 4, art. 72.
9 out of No. 2, art. 71.
10 out of No. 1, art. 71.

All these pieces, except Nos. 5 and 6, were rested in triangular saddles of hard wood, cut very exactly to the angle of the batten, when they were broken with their edge down; but when the edge was upward, a similar one was placed on the centre, in order that the weight might not break down its edge. This latter saddle was about half an inch thick.

Nos. 5 and 6 had pieces glued and screwed on at their ends, in order to render their bearings solid; but it did not appear to make any difference: they were weaker than Nos. 3 and 4; but the piece from which they were made, viz. No. 4, Art. 72, was itself very weak, as appears by that experiment.

TABLE VII.

(77.) *Experiments on Fir Battens, fixed at each End.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Deflection.	Weight in lbs.	Reduced to Sp. Gr. 600.	REMARKS.
1	72	2	2	581	·45 1·00 1·30 2·1	220 620 822 1024	1058	The whole time of the experiment 34 min.; after last weight 6 min.
2	72	2	2	581	·41 ·95 1·25 2·1	220 620 822 1139	1174	Whole time 28 m.
3	72	2	2	611	·40 ·87 1·35 2·2	220 620 822 1090	1070	Whole time 45 m.
4	72	2	2	600	·45 1·00 1·20 2·3	220 620 822 1120	1120	Whole time 18 m.
					Mean	Weight	1105	

Nothing remarkable occurred in making these experiments. We have before (Art. 65) explained the methods that were employed in order to ensure a permanent fixing of the two ends, which was done with the greater care, as experiment and theory differed very materially in the comparative strength of equal battens, when *fixed* at each end, and when only *supported*: all former theories make the strength in the two cases as two to one, while most experimentalists state it as in the ratio of 3 : 2. According to the former, the mean strength of these beams, as compared with those at Art. 72, ought to have been 1442 lbs., and according to the latter, 1116 lbs.: the mean is 1105 lbs.: and it is shewn, in a subsequent page, that the correct theory gives about the same result.

TABLE VIII.

(78.) *Experiments on Fir Battens, fixed at one End, at different Angles of Inclination, and in different Positions.*

No. of Experiments.	*Length in Inches.*	*Depth in inches.*	*Breadth in inches.*	*Specific Gravity.*	*Weights in lbs.*	*Deflection in inches.*	*Weight reduced to Length 36, and Sp. Gr. 600.*	*Position of the Beams, &c.*
1	36	2	2	560	317	5·0	400	Side parallel to the horison.
2	32	2	2	609	432	6·0	400	
3	32	2	2	571	417	6·0	389	
4	30	$\sqrt{8}$	$\sqrt{8}$	600	462	4·9	385	Diagonal vertical.
5	30	$\sqrt{8}$	$\sqrt{8}$	613	469	4·7	391	
6	30	$\sqrt{8}$	$\sqrt{8}$	620	466	4·9	389	
7	24	2	1	620	279	4·1	180	Horizontal.
8	24	2	1	600	276	3·9	184	
9	24	2	1	596	273	4·3	183	Angle 26° upwards.
10	24	2	1	581	281	4·1	193	
11	24	2	1	600	294	3·9	196	Angle 26° downwards.
12	24	2	1	601	290	4·0	193	

The first six of the above pieces were the fragments of the first two and last specimens of the preceding page; care having been taken, in those experiments, to prevent the weights from going quite down, which would have endangered the breaking of the pieces at the ends where they were fixed in the wall. By blocking the scale as soon as the fracture commenced in the middle, the ends were left perfectly whole, the parts recovering completely their original rectilinear form.

The first three of the above were broken in the same position; viz. with the sides parallel and perpendicular to the horizon; the next three angle-ways, viz. with the diagonal vertical.

Nos. 7 and 8 were fixed in the usual horizontal position; Nos. 9 and 10, which were the same two pieces inverted, or turned end for end, were fixed at an angle of inclination upwards of 26°; and Nos. 11 and 12 at the same angle downwards.

TABLE IX.

EXPERIMENTS

(79.) *On Oak Battens, supported at each End.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Deflection.	Weights in lbs.	Reduced to Sp. Gr. 800.	Mean Reduced Weight.
1	18	1	1	767	—	323	337	358
2				768	—	353	368	
3				768	—	339	368	
4	24	1	1	764	—	266	278	269
5				774	—	251	260	
6				774	—	260	268	
7	30	1	1	777	—	196	202	202
8				777	—	196	202	
9				777	—	196	202	
10	36	1	1	—	2·95	158		*180
11				—	4·20	190		
12				—	—	176		

Nos. 1, 2, 5, and 4, were all from one piece, near the root end, and rather cross-grained, particularly Nos. 1 and 5. Nos. 2 and 4 were cut from the ends of these. Nos. 7, 8, and 9, each bore 286 lbs. without any appearance of fracture; but each broke immediately with the addition of 15 lbs.: it was, therefore, only taken as 10 lbs.

No. 11 was remarkably elastic; and, just before its fracture, its curve was traced on a plane-board placed against it, and the ordinates carefully measured at every inch, and were found as follow:

Ordinates,	·26,	·53,	·85,	1·13,	1·4,	1·7,	1·93,	2·2,	2·45.
Abscisses,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Ordinates,	2·65,	2·87,	3·1,	3·3,	3·46,	3·63,	3·75,	3·82,	3·9.
Abscisses,	10,	11,	12,	13,	14,	15,	16,	17,	18.

* The specific gravities of 10, 11, 12, were not noted; the mean 180 is found by assuming them at 777, being part of the same plank as the above.

TABLE X.

EXPERIMENTS

(80.) *On Oak Battens, supported at each End.*

No. of Experiments.	*Length in inches.*	*Depth in inches.*	*Breadth in inches.*	*Specific Gravity.*	*Deflection.*	*Weight in lbs.*	*Reduced to Sp. Gr. 800.*	*Mean Reduced Weight.*
1	24	$1\frac{1}{2}$	$\frac{3}{4}$	768	1·1	387	403	408
2				784	1·1	408	416	
3				777	1·1	395	406	
4	30	$1\frac{1}{2}$	$\frac{3}{4}$	777	1·5	316	325	326
5				784	1·5	327	333	
6				768	1·5	300	311	
7	30	2	1	777	1·4	721	742	753
8	30	2	1		1·4	736	758	
9	30	2	1		1·4	736	758	
10	36	2	1	764	—	598	626	634
11					—	607	635	
12					—	612	641	

The successive deflections of Nos. 1, 2, 3 were measured as follow; viz.

Weights.	*Deflection of No. 1.*	*No. 2.*	*No. 3.*
321	·65	·62	·65
366	·85	·72	·85
380	1·05	·95	1·05
387	1·1	1·05	1·08

The deflections of Nos. 7 and 9 were exactly equal, and were measured on three equidistant ordinates: the lengthening of the fibres were also in both cases equal: the particulars are as below; viz.

Weights.	*Deflections.*			*Lengthened.*
421 Nos. 7 and 9,	·1	·25	·366	·075
521	·13	·35	·466	·100
621	·19	·50	·700	·125
671	·20	·60	·800	·150
721	..	..	1·40	

The deflections of No. 8 were not observed.

TABLE XI.

EXPERIMENTS

(81.) *On Ash Battens fixed at one End in a Wall.*

No. of Experiments.	*Length in inches.*	*Depth in inches.*	*Breadth in inches.*	*Specific Gravity.*	*Deflection.*	*Weight in lbs.*	*Weight reduced to 36 inches length, and 2 in. square.**	*Position of the Beams.*
1	36	2	2	658	$11\frac{1}{2}$	436	436	Side parallel to the horizon.
2	36	2	2	730	$14\frac{1}{2}$	431	431	
3	30	$\sqrt{8}$	$\sqrt{8}$	658	$5\frac{1}{4}$	471	392	Diagonal vertical.
4	30	$\sqrt{8}$	$\sqrt{8}$	730	$6\frac{3}{4}$	466	388	
5	24	2	1	730	5	352	470	Fixed at an ∠ 26° down.
6	24	2	1	730	not obs.	321	428	Ditto ditto upwards.
7	24	2	1	730	6	332	441	Horizontal.
8	24	2	1	730	6	321	428	Horizontal.
9	24	2	1	730	not obs.	302	403	Angle 26° upwards.

No. 1 was the same piece as No. 3.
No. 2 — the same piece as No. 4.
No. 5 — the same piece as No. 7.
No. 8 — the same piece as No. 9.

Nos. 1, 2, 5, and 8 were first broken at one end, (but not so as to completely separate the parts); after which they were turned end for end, and broken again, as stated in Nos. 3, 4, 7, and 9. No. 6 was so fractured in the first experiment, that it could not be submitted to a second trial. The same thing always occurred when the beam was first broken at an angle upwards: it appeared, in these cases, to turn on a point about 6 inches from the wall where the strain and curvature seemed to be the greatest, and from which point the fracture commenced, splitting the piece through its whole length.

In the above experiment, No. 2, the neutral line was remarkably well defined, and appeared to be very nearly, or exactly, at $\frac{5}{8}$th of the whole depth; the same as in fir.

* The reduction in column 8 is made on a supposition that the strength is inversely as the length.

TABLE XII.

EXPERIMENTS

(82.) *On Beech Battens fixed at one End in a Wall, at different Inclinations and in different Positions.*

No. of Experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific Gravity.	Deflection.	Weight in lbs.	Weight reduced to 36 inches length, and 2 inches square.	Position of the Beam.
1	36	2	2	700	11	401	401	Side parallel to the horizon.
2	36	2	2	690	8	401	401	
3	36	2	2	700	11	401	401	
4	30	$\sqrt{8}$	$\sqrt{8}$	690	5	466	388	Diagonal vertical.
5	30	$\sqrt{8}$	$\sqrt{8}$	700	6	451	376	
6	24	2	1	740	$4\frac{1}{2}$	371	495	Fixed at an ∠ 26° down.
7	24	2	1	740	5	352	469	Ditto ∠ 26° upwards.
8	24	2	1	740	5	352	469	Horizotal.
9	24	2	1	740	5	352	469	Ditto.
10	24	2	1	740	$5\frac{1}{2}$	317	463	At an ∠ 26° upwards.

No. 2 was the same piece as No. 4.
No. 3 — the same piece as No. 5.
No. 6 — the same piece as No. 8.
No. 9 — the same piece as No. 10.

Nos. 1 and 7 were so much splintered in the first experiments, that they could not be submitted to a second trial, as was done with Nos. 2, 3, 6, and 9. These, after being broke at one end, (without a total separation), were turned end for end, and then broken with the weights indicated in Nos. 4, 5, 8, and 10.

It should be observed here, that the deflections were not, in these experiments, measured so accurately as in those that were supported at each end: the apparatus not being so convenient, we were generally satisfied with measuring it to the nearest $\frac{1}{4}$ of an inch: the successive deflections, however, seemed to follow, while the weights were small, the ratio of the weights, as was observed in the preceding experiments. The deflections from first to last were as follow:

121 lb.	No. 1 = $1\frac{7}{8}$	No. 2 = $1\frac{9}{16}$	No. 3 = $1\frac{7}{8}$
221 lb.	= $3\frac{5}{8}$	= 3	= $3\frac{3}{8}$
271 lb.	= 5	= $3\frac{7}{8}$	= $4\frac{1}{2}$
321 lb.	= 7	= 5	= $6\frac{1}{8}$
401 lb.	= 11	= 8	= 11

TABLE XIII.

(83.) *Experiments on Solid and Hollow Cylinders, supported at each End.*

No. of Experiments.	*Names of Woods.*	*Specific Gravity.*	*Length in inches.*	*Diameter external.*	*Diameter internal.*	*Breaking weight.*	*Deflections in inches.*
1	Fir.	581	48	2	solid	740	2·0
2		603	48	2	do.	796	2·1
3		580	48	2	do.	780	1·9
4	Ash.	590	46	2	solid	700	2·7
5		590	46	2	solid	730	2·5
6		586	46	2	$\frac{1}{2}$ inch	650	3·0
7		540	46	2	$\frac{1}{2}$ inch	664	3·0
8		601	46	2	$\frac{3}{4}$ inch	646	3·1
9		601	46	2	$\frac{3}{4}$ inch	654	2·9
10		580	46	2	1 inch	631	2·8
11		580	46	2	1 inch	630	3·6

The fir pieces were part of the same plank as those of 4 feet, given at Art. 71, viz. Nos. 3 and 4, which was a very fine specimen of Christiana deal, and had been in store a considerable time.

The ash cylinders were obviously of a much weaker quality than those of which the detail is given at Art. 81; but the results were very uniform, and they, therefore, furnish a good comparison between the strength of solid and hollow cylinders amongst themselves, although we cannot compare them with our square battens, as they were of a much inferior quality to the preceding square pieces. The fir cylinders, on the contrary, furnish no comparison between solid and hollow cylinders; but they may be correctly compared with like pieces of the same dimension square, being, as stated above, precisely the same wood as Nos. 3 and 4, Art. 71.

84. Similar experiments to those last described were made on battens of elm and teak; but the results of the latter were so irregular; that it would be useless to give the detail of them: it will be sufficient to observe, that one of the pieces of teak bore 478 lbs., which was more than equal to the load borne by the ash pieces of the same dimensions; viz. 3 feet long by 2 inches square; while the other two pieces broke with little more than 300 lbs., the deflection in each case being about 7 inches: and one piece 2 feet long, 2 inches deep, and 1 inch in breadth, fixed at one end, and at an angle of 26° upwards, broke with 422 lbs., which is considerably more than was found to be necessary for breaking an equal piece of ash.

The elm battens gave much more uniform results, although the pieces were found very weak in comparison with those of ash and beech. The mean weight which broke the three pieces 3 feet long and 2 inches square, was 216 lbs.; and the mean of the same three pieces inverted and fixed diagonally, was 296 lbs., the latter being broken at 30 inches; the mean specific gravity was 570.

Remark.—If the same reduction be made here as in the pieces of ash and beech, we shall have

$$36 : 30 :: 296 : 246,$$

which shews that the strength of elm is the same whether it be fixed direct or diagonally; whereas it was found that ash and beech were both weakest in the latter position.

AN ESSAY

ON THE

STRENGTH AND STRESS OF TIMBER.

PART III.

ON THE DEFLECTION OF BEAMS, WHEN EXPOSED TO A TRANSVERSE STRAIN.

Of the Deflection in Terms of the Length.

85. It has been shewn in the preceding pages, that according to the hypotheses of Galileo and Leibnitz, a beam of timber, when fixed solidly in a wall at one end, and loaded with a weight at the other, will turn about the lowest point of the section of fracture; as, for example, the line CD, *fig*. 1, *plate* I: but it has also been explained that this supposition is erroneous; and some experiments have been referred to, which shew clearly that the rotation, instead of being made about the lowest point, as supposed by these authors, is performed about a certain line within the section of fracture; and, therefore, only those fibres which are above it are exposed to tension, and those below to compression; while that thin plate or lamina of them, which coincides with its plane, is neither extended nor compressed: for which reason, this line may be properly denominated the *neutral axis of rotation*.

Thus, in *fig.* 5, where AFIC represents the section of a beam fixed, and loaded as supposed above, *n* denotes the section of the neutral axis; A*nb* the quantity of extension, and *anc* the quantity of compression, to which the fibres above and below that line are exposed previous to the fracture taking place.

In order to establish a correct theory of the strength and stress of timber, it is of the greatest importance to know, very accurately, the situation of the neutral axis; but it is not essential to our present inquiry, which is not to determine the absolute quantity of deflection, but the proportional quantity of it, as depending upon the length, depth, breadth, and position of the beam.

86. In order to this, let ABCD, *fig.* 6, represent a beam fixed into a solid wall, and in its natural horizontal position, its weight being supposed nothing, or inconsiderable with regard to that with which it is loaded: and let us suppose it to be made up of the several lamina AB*ab*, $aba'b'$, $a'b'a''b''$, &c., each of which is considered to be subject to compression and extension: then, when the beam is loaded with a weight W, it will be brought into the curvilinear form shewn in the second position in the figure. Draw the several tangents A*m*, *an*, $a'o$, $a''p$, &c.; and admitting that the quantity of extension and compression is proportional to the extending and compressing forces, we shall have the several angles *man*, noa', $oa''p$, pa'''D, &c. as the distances CF, *Cf*, Cf', Cf'', &c., these being the effective lengths of the levers, by means

of which the force or weight W is exerted at those several points: and the same will have place if we suppose the number of lamina to be indefinitely great, and therefore the thickness of each indefinitely small: and hence we see the fundamental property of the curve which a beam thus fixed and loaded will assume; viz. "that the curvature at every point is as the distance of that point from the line of direction of the weight," which is, in fact, the *elastic curve*, first proposed by Galileo, but the correct investigation of which we owe to James Bernoulli, who published it in the Memoirs of the Academy of Sciences for 1703. Other investigations of it have since been given by John Bernoulli, "Opera Omnia," tom. iv. p. 242; as also in his Essay on the Theory and Manœuvres of Ships; and particularly by Euler, in the appendix to his celebrated work, "Methodus inveniendi Lineas Curvas."

87. It is to be observed, however, that the supposition of the extension and compression being exactly proportional to the exciting forces, is only a particular and very limited case of the elastic curve; for if that extension were as any function of those forces, it would still not wholly change, although it would modify, the fundamental property of it: but its investigation under this general character would carry us far beyond our present purpose, and, at the same time, would be of no use in our future investigation; for it appears from experiment, that the quantity of extension, in consequence of the imperfect elasticity of the fibres, is very irregular, and that after a certain deflection

has been obtained, it seems subject to no determinate law; a circumstance which we have endeavoured to illustrate in a subsequent article: but during the early part of the experiment, that is, while the weight is considerably less than that which is required to produce the ultimate fracture, the law of the deflections is nearly uniform, and proportional to the exciting force; it will, therefore, be sufficient to consider the elastic curve under this particular case, being the only one that is applicable to the present inquiry.

Let, then, A B, *fig.* 5, *plate* III. represent a thin elastic lamina, without weight, and in its first natural horizontal position; A C the position of it after being loaded with any given weight W: at any point in the curve R draw the tangent R T, and conceive the curve to be divided into an indefinite number of equal small parts A a, R r; and since, by the hypothesis, the extension of each fibre is proportional to the force by which it is excited; if $r\,s$ and $b\,a$ be drawn perpendicular to the curve at a and r, the former may be taken to denote the extension of the particle A a, and the latter that of the particle R r; and we shall have $r\,s : a\,b ::$ force in R : force in A, or $:: \mathrm{C\,L} \times \mathrm{W} : \mathrm{C\,G} \times \mathrm{W}$. Let A F and R X be the radii of curvature at the points A and R, then the triangles A a b and A a F, as also R s r and R r X are similar; and, therefore, since A a = R r, we have

$$r\,s : \mathrm{R}\,r :: \mathrm{R}\,r : \mathrm{R\,X}$$
$$a\,b : \mathrm{A}\,a :: \mathrm{A}\,a : \mathrm{A\,F}$$
$$\text{therefore } r\,s : a\,b :: \mathrm{A\,F} : \mathrm{R\,X}$$
$$\text{but } r\,s : a\,b :: \mathrm{C\,L} : \mathrm{C\,G}$$

and consequently

$$CL : CG :: AF : RX$$

whence again $CL \times RX = CG \times AF$, a *constant quantity* $= A$.

In order now to trace the property of the curve, let $CL = x$, $RL = y$, and $RC = z$; then, as is shewn by writers on fluxions, the radius of curvature

$$RX = \frac{\dot{z}^3}{-\dot{x}\ddot{y}} = \frac{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{-\dot{x}\ddot{y}}$$

and consequently

$$\frac{x\dot{z}^3}{-\dot{x}\ddot{y}} = A, \text{ or } \frac{x(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{-\dot{x}\ddot{y}} = A.$$

In its present form this equation is not integrable, except by means of series; but we may accommodate it to our purpose, without any sensible error, while the deflections are small, by supposing $\dot{x} = \dot{z}$, in which case it becomes

$$\frac{x\dot{x}^2}{-\ddot{y}} = A, \text{ or } x\dot{x} = A\frac{\ddot{y}}{-\dot{x}}$$

Or assuming $\dot{x}$ as constant, and taking the fluent

$$\tfrac{1}{2}x^2 + C = \frac{-A\dot{y}}{\dot{x}}, \text{ C being the correction.}$$

Now, when $x = l$, $\frac{1}{2}l^2 + C = o$, $\frac{\dot{y}}{\dot{x}}$ being in that case $= o$, therefore the correct fluent is

$$\tfrac{1}{2}(x^2 - l^2) = -A\frac{\dot{y}}{\dot{x}}$$

Multiplying now by $\dot{x}$, we have

$$\tfrac{1}{2}\dot{x}(x^2 - l^2) = -A\dot{y},$$

and taking the fluent

$$\tfrac{1}{2} l^2 x - \tfrac{1}{6} x^3 = A y,$$

which requires no correction.

By means of this equation, the curve may be constructed while the deflections are small with regard to the length of the lamina; but it will obviously apply to no other case, because it is obtained on a supposition of $\dot{x}$ being equal to $\dot{z}$, which is in no case strictly true; although the difference, while the deflections are small, is inconsiderable, and may be admitted without any sensible error.

Writing l for x and b for y, the above becomes

$$\frac{l^3}{b} = A, \text{ or } \frac{l^3}{b} = C G \times A F:$$

or, since in the case here supposed $C G = l$ *very nearly*, this equation may be still farther reduced to

$$\frac{l^2}{b} = A F:$$

and hence it follows, that while A F remains constant, or the curvature at A is the same; that is, while the strain upon the beam at that point is constant, the deflection b must vary as the square of the length.

But the strain (the weight remaining the same) is as l; therefore A F is reciprocally as l: and, therefore, while the weight is the same,

$$\frac{l^2}{b} = \frac{A F}{l} \text{ or } \frac{l^3}{b} = A F:$$

consequently, while the weight remains the same, the deflection b is as the cube of the length: but we have seen that, *cæteris paribus*, the deflection is as the weight; therefore, generally

$$\frac{W l^3}{b} = E, \text{ a constant quantity;}$$

that is, the deflection is as the weight into the cube of the length.

88. This is contrary to the experimental results of M. Girard, and ought to be examined with particular care: we propose, therefore, investigating the nature of the curve on different principles, and on such as will probably be more intelligible to many readers.

It has been shewn above, that an approximation to the actual state of the curve is all that can be obtained; but this approximation may, we conceive, be found much more satisfactorily, as follows.

Let ABCD, *fig.* 6, *pl.* I. represent the deflected beam, and let it be divided as above supposed (Art. 86.) into any number of equal inflexible lamina, A B $a\ b$, $a\ b\ a'\ b'$, &c., and let $a\ d$, $a'\ d'$, $a''\ d''$, &c. drawn perpendicular to the respective tangents at A, a, a', &c. represent the deflections at those points, and which, from what has been above shewn, will be proportional to C F, C f, C f', &c.; and as the investigation is only intended to apply to small deflections, let us consider these several lines, $a\ d$, $a'\ d'$, &c. instead of being perpendicular each to its respective tangent, to be all parallel to each other, and perpendicular to A m: let us also denote the first of these $a\ d$ by d, which may be denominated the *element of deflection*, and let the number of parts or lamina into which the beam is divided be denoted by m, then we shall have

$$\text{---------------}\ \frac{m}{m}d = a\,d$$

$$m : m-1 :: d : \frac{m-1}{m}d = a'\,d'$$

$$m : m-2 :: d : \frac{m-2}{m}d = a''\,d''$$

$$m : m-3 :: d : \frac{m-3}{m}d = a'''\,d'''$$

&c. &c. &c.

Also, according to our supposition,

$$n\,m = m \times a\,d = \frac{m^2}{m}\,d$$

$$n\,o = (m-1)\,a'\,d' = \frac{(m-1)^2}{m}\,d$$

$$o\,p = (m-2)\,a''\,d'' = \frac{(m-2)^2}{m}\,d$$

&c. &c. &c.

Whence the whole deflection m D will be expressed by the series

$$m\,\mathrm{D} = \frac{d}{m}\left\{m^2 + (m-1)^2 + (m-2)^2 + \&c.\ 1^2\right\}\ldots\ldots(1)$$

or by the summation of the series,

$$m\,\mathrm{D} - \frac{d}{m}\left\{\frac{m^3}{3} + \frac{m^2}{2} + \frac{m}{6}\right\}, \text{ or}$$

$$m\,\mathrm{D} = d\left\{\frac{m^2}{3} + \frac{m}{2} + \frac{1}{6}\right\}$$

That is, while the number of parts m are supposed finite, m D varies as $\left(\frac{m^2}{3} + \frac{m}{2} + \frac{1}{6}\right)d$; but when m is infinite, then the two latter terms vanish, as being inconsiderable with regard to the first; and we have $m\,\mathrm{D} = \frac{m^2\,d}{3}$.

In the same manner, if l' were the length of any other beam, of which the number of parts were m', but in length equal to the former, and the

element of deflection $\acute{d}$, we should have $\acute{m}\ \acute{D} = \frac{\acute{m}^2\,\acute{d}}{3}$;

Whence $m\,D : \acute{m}\,\acute{D} :: m^2\,d : \acute{m}^2\,\acute{d}$; but $m : \acute{m} :: l : \acute{l}$; therefore, $m\,D$ varies as $\frac{l^2\,d}{3}$*;

that is, the deflection varies as the square of the length, and the element of deflection; but the element d obviously varies as the strain; that is, as $l\,W$: therefore again the deflection varies as $\frac{l^3\,W}{3}$; or denoting the deflection $m\,D$ by b, we have $\frac{l^3\,W}{3\,b} = E$, a constant quantity, the same result as before.

89. The same may be otherwise demonstrated as follows:

In the above investigation it is shewn that $D\,m$, which is supposed to represent the deflection, is expressed by the equation

$$m\,D = d\left\{\frac{m^2}{3} + \frac{m}{2} + \frac{1}{6}\right\}$$

and that in any other beam, of which the number of parts are $\acute{m}$, the deflection is also

$$\acute{m}\,\acute{D} = \acute{d}\left\{\frac{\acute{m}^2}{3} + \frac{\acute{m}}{2} + \frac{1}{6}\right\}$$

* We have used the above process for the convenience of those who may not be acquainted with the fluxional analysis: those who are, will see immediately that the summation, expressed in equation (1), is equal to $\frac{d}{m}$ times the integral of $x^2\,\dot{x}$; that is,

$$\frac{d}{m}\int x^2\,\dot{x} = \frac{d\,x^3}{3\,m} = \frac{d\,m^2}{3} \text{ when } x = m.$$

from which we conclude, that when m is infinite, the deflections are as

$$d\,m^2 : d'\,m'^2;\ \text{or as}\ d\,l^2 : d'\,l'^2;$$

where l and l' denote the two lengths. If this should not appear to involve all that precision and accuracy that may be desired, it may be considered under a point of view somewhat different to the former, and will probably carry more conviction with it to some of our readers:

Supposing, therefore, the equation

$$m\,D = d\left\{\frac{m^2}{3} + \frac{m}{2} + \frac{1}{6}\right\}$$

to be established; and calling l the length of the beam, and λ the length of each of the equal sides of the polygon, we shall have $\frac{l}{\lambda} = m$; and substituting this for m in the preceding equation, we obtain

$$m\,D = d\left\{\frac{l^2}{3\,\lambda^2} + \frac{l}{2\,\lambda} + \frac{1}{6}\right\}, \text{ or}$$

$$m\,D = d\left\{\frac{2\,l^2 + 3\,l\,\lambda + \lambda^2}{6\,\lambda}\right\};$$

and in the same manner, if the length of another beam is l', and $m'\,D'$ denotes its deflection, we find

$$m'\,D' = d'\left\{\frac{2\,l'^2 + 3\,l'\,\lambda + \lambda^2}{6\,\lambda^2}\right\};$$

λ, or the length of each side of the polygon, being, by the supposition, the same in both cases; we shall have, therefore,

$$m\,D : m'\,D' :: d\left\{2\,l^2 + 3\,l\,\lambda + \lambda^2\right\} : d'\left\{2\,l'^2 + 3\,l'\,\lambda + \lambda^2\right\}.$$

This result is wholly independent of any particular value of λ, and therefore is true, when λ becomes indefinitely small; that is, in the case of a

continued curve. But here, as λ is indefinitely small, the last two terms of each of the third and fourth members of the above ratio vanish, and that ratio then becomes simply

$$m\,D : \acute{m}\,\acute{D} :: d\,l^2 : \acute{d}\,\acute{l}^2;$$

that is, the deflection varies as the element of deflection into the square of the length; or, as the element of deflection into the square of the length divided by 3, as we have found it in the article in question.

90. In a similar manner we may investigate the law of deflection when the weight, instead of being all applied at the extremity of the beam, is equally distributed throughout its whole length, or when it is divided into equal portions, and suspended at equal distances, as at the points $\acute{a}$, $\acute{\acute{a}}$, $\acute{\acute{\acute{a}}}$, &c. fig. 6.

For calling $\acute{d}$, as before, the element of deflection $= a\,d$, it is obvious that the successive deflections, instead of decreasing as before, in the simple ratio of the length, will now decrease as the square of the length, because both the weight and the length of lever decrease in the same manner. Our successive deflections, therefore, in this case, will be

$$\frac{m^2}{m^2}\acute{d} = a\,d$$

$$m^2 : (m-1)^2 :: \acute{d} : \frac{(m-1)^2}{m^2}\,\acute{d} = \acute{a}\,\acute{d}$$

$$m^2 : (m-2)^2 :: \acute{d} : \frac{(m-2)^2}{m^2}\,\acute{d} = \acute{\acute{a}}\,\acute{\acute{d}}$$

$$m^2 : (m-3)^2 :: \acute{d} : \frac{(m-3)^2}{m^2}\,\acute{d} = \acute{\acute{\acute{a}}}\,\acute{\acute{\acute{d}}}$$

&c. &c. &c.

Also, according to the same supposition as that above adopted, we shall have

$$n\,m = m \times a\,d = \frac{m^3}{m^2}\,d'$$

$$n\,o = (m-1)\,a'\,d' = \frac{(m-1)^3}{m^2}\,d'$$

$$o\,p = (m-2)\,a''\,d'' = \frac{(m-2)^3}{m^2}\,d'$$

&c. &c. &c.

Whence the whole deflection m D will now be expressed by the series

$$m\,\mathrm{D} = \frac{d'}{m^2}\left\{m^3 + (m-1)^3 + (m-2)^3 + \text{\&c.}\, 1^3\right\}$$

or by summation,

$$m\,\mathrm{D} = \frac{d'}{m^2}\left\{\frac{m^4}{4} + \frac{m^3}{2} + \frac{m^2}{2}\right\};\ \text{or,}$$

$$m\,\mathrm{D} = d'\left\{\frac{m^2}{4} + \frac{m}{2} + \frac{1}{4}\right\}:$$

which expression is analogous to that in Article 88, and shews that in this case also, when m is infinite, that is, when the weight is uniformly distributed, the deflection is as the weight and cube of the length, because the expression then becomes

$$m\,\mathrm{D} = \frac{m^2}{4}\,d'.$$

But in order to compare the real quantity of deflection in this case with that of the former, it must be observed, that the weight being the same, the strain on the beam will, in the first instance, be double what it is in the second; and the element d in the former will be double d' in the latter, or $d' = \frac{1}{2}d$. Substituting, therefore, $\frac{1}{2}d$ for d', our expression m D

$= \frac{m^2}{4} d$, becomes $\frac{m^2}{8} d$; whereas in the former case it is $\frac{m^2}{3} d$; therefore the beams being of the same length, the deflection, when the weight is all collected at the extremity, is to that of the beam equally loaded throughout its length with the same weight, as $\frac{m^2}{3} d : \frac{m^2}{8} d$, or as 8 to 3.

The expression for the elasticity in this case will therefore be $\frac{l^3 W}{8\, b} = E$, the same constant quantity as before.

The principles of investigation given in Art. 89 are equally applicable in this case.

91. In the preceding investigations, we have considered the deflection only with reference to beams fixed at one end: let us now endeavour to investigate the same, on a supposition of their being supported at both ends. In order to which, it may be observed, in the first place, that whatever weight is just sufficient to break a beam fixed by one end in a wall, the same weight may be borne at the other end of it (the arms or levers being supposed of equal length), if the wall were removed, and the beam merely supported on a fulcrum, or prop, in its middle point, as in *fig.* 8, *plate* II. the tension in both cases being the same; just as a line passing over a pulley, and loaded at each end with an equal weight, has the same tension as a single fixed line, loaded with only one of those weights: and what is here stated of the ultimate degree of tension, is obviously true of any quantity of it: that is, whatever tension the fibres may have in the former

case, they will have precisely the same in the latter. But it is not the same with the deflections under these two circumstances of equal strains, the element of deflection being, in one case, double that in the other. For the extension of the fibres A *b*, A *b*, *fig.* 5 and *fig.* 8, being equal by the supposition, the angles A *n b*, in both figures, will be equal: but as in one (*fig.* 5) the line *n b* is vertical, and in the other (*fig.* 8), it declines equally from the vertical with the line *n* A, the deflection of the beam (supposing it, for simplicity, to remain inflexible in every part, except in the section A *n* C), in the latter case, from the line H H, will be only half that of the half beam fixed as in *fig.* 5; that is, the element of deflection in the former instances will be only half that in the latter; and consequently, as we have shewn that the deflections are, *cæteris paribus*, as the element of deflection, it follows, that the successive and ultimate deflections in the two cases will have the same ratio; that is, they will be to each other as 2 to 1.

Again, the beam $F\ I\ F'\ I'$, *fig.* 8, is similarly situated, at least as far as our present question is concerned, with regard to the strain upon it, and therefore to its deflections, as the equal beam $F\ I\ F'\ I'$, *fig.* 9; whether we consider the latter to rest against a fulcrum at C, and to be strained by the two weights W, W' passing over the pulleys Q, Q'; or, as being supported on two fulcrums, F, F', and loaded in the middle with the weight P, equal to the two weights W, W'.

Hence, then, we conclude, that the deflection of a beam fixed at one end in a wall, and loaded at the other, is double that of a beam of twice the length, supported at both ends, and loaded in the middle with a double weight; that is, the strain being the same in both cases: consequently, when the weights are the same, the deflection in the first instance is to that in the second as 4 : 1.

And when the length and weight are both the same, the deflections will be to each other as 1 : 32.

For the strain will be four times greater on the beam fixed at one end than on that supported at both; and, therefore, all other things being the same, the element of deflection would also be four times greater: but we have seen, that with the same strain the element of deflection is double in one case what it is in the other: it will, therefore, by combining the two effects, be 8 times greater: also, the entire deflection is as the element of deflection into the square of the length; and, according to our supposition, the length is double; whence, upon the whole, it appears that the deflection in the one case is to that in the other as 1 : 8 × 4, or as 1 to 32.

The same formula will, therefore, apply in this case as in Art. 87; viz. $\frac{l^3 W}{3 \delta} = E$, a constant quantity; observing only, that the value of E is here 32 times greater than in the former.

92. When the weight is distributed throughout the length of the beam, instead of being all collected in the middle, it is a known mechanical

principle, that the strain on the centre will be the same as it would be with half the entire weight collected in that point; and, consequently, the element of deflection in the same place will also be one-half of what it would be if the whole weight was collected there.

But now, in order to compare the strain and consequent deflection at any other point, D, *fig.* 10, *plate* I. we must first observe, that the resistance of the fulcrum at B is constant; and therefore, that the strain at D, as arising from that resistance, will be found as follows; viz. C B : D B : $d' : \frac{\text{D B}\, d'}{\text{C B}}$ = the element of deflection at D, as arising from the resistance at B; d' denoting the deflection at C.

But the point D has a farther strain to sustain, and consequently a farther deflection, arising from the weight of the part between C and D. Now this weight will be to the whole weight W, as C D to A B, or 2 C B; that is,

$$2\,\text{C B} : \text{C D} :: \text{W} : \frac{\text{C D} \times \text{W}}{2\,\text{C B}}.$$

Consequently, the deflection arising from this strain, as referred towards B, will be

$$\text{C B}^2 : \text{C D} \times \text{B D} :: d' : \frac{\text{C D} \times \text{B D}}{\text{C B}^2}\, d'.$$

Whence the entire deflection from the tangent of the curve at the point D will be

$$\frac{\text{D B}}{\text{B C}}\, d' + \frac{\text{C D} \times \text{D B}}{\text{B C}^2}\, d' = \frac{(\text{C B} + \text{C D})\, \text{D B}}{\text{B C}^2}\, d'.$$

Which deflection referred to the perpendicular B F, will be $\frac{(\text{C B} + \text{C D})\, \text{D B}^2\, d'}{\text{B C}^3}$.

If, now, we denote C B by m, and D B by n, in which case C D $= m - n$, the above will become

$$\frac{(2m - n)n^2}{m^2} d' = \frac{2mn^2 - n^3}{m^2} d'.$$

And, by giving to n the successive values, 1, 2, 3, &c. as in our preceding investigations, and summing the resulting series, or by finding the value of

$$\int \frac{2mx^2 - x^3}{m^2} d' \dot{x}$$

when $x = m$, we shall have for the entire deflection,

$$\text{B C} = \frac{5m^2}{12} d.$$

But it has been shewn, that in the former case, where the weight is all collected in the middle, the deflection is $\frac{m^2}{3} d$; and, therefore, since $d' = \frac{1}{2} d$, the deflections in the two cases will be as $\frac{1}{3} : \frac{5}{24}$, or 8 to 5.

Now we have seen, that when a beam or rod is fixed only at one end, the deflection, when the weight is uniformly distributed, is to the same when that weight is collected at the extremity, as 3 to 8: whereas we have found above, that when the beam is supported at its ends, the deflections in the like cases are to each other as 5 to 8.

Whence, if a long rod or plank is, in the first instance, supported in the middle, and the ends be deflected; and, in the second, the ends are supported, and the middle left to descend, the deflection in the latter case is to that in the former as 5 to 3.

Of the Deflection in Terms of the Breadth and Depth.

93. In the preceding investigations we have supposed the beams, although of different lengths, to be all of the same breadth and depth; or, as opposing equal resistance: when these dimensions are not the same, the resistance is as the breadth and square of the depth;* and, therefore, when the weight is increased in that proportion, the quantity of extension will, by hypothesis, be the same, the length being here supposed constant: but, by a reference to *fig.* 5, *plate* II. it will appear, that the extension of the fibre b A being supposed constant, the angle $b\,n$ A, or H A F, (which is equivalent to what we have denominated the element of deflection,) will be reciprocally as n A, or C A, the depth of the beam: whence it follows, that in beams of the same breadths but of different depths, when the strain is proportional to the resistance, the element of deflection will be inversely as the depth; and hence, by combination, the deflection is directly as the weight, and inversely as the breadth and cube of the depth, the length being the same: but we have seen, that when the breadth and depth are constant, the deflections are as the cubes of the length; therefore, generally, if l be the length of any beam, a its breadth, d its depth, and W the weight with which it is loaded, the deflection will

* This has not been demonstrated in any of the preceding articles; and the reader may, therefore, either refer to the investigations in the following chapter, or he may make his deduction from the foregoing experiments, which all indicate the above ratio.

vary as $\frac{l^3 \times W}{a \times d^3}$; and if, therefore, we denote the deflection by δ,

$$\frac{l^3 W}{a d^3 \delta} = E, \textit{ a constant quantity.}$$

94. This is a conclusion which necessarily arises out of the above investigation, but it is at variance with the theory and experiments of M. Girard; and as these latter are very numerous, and as they were conducted on a large scale, and the work containing them being published under the sanction of the French National Institute, I was a little surprized at the result thus obtained, and re-examined my investigations, under an impression that some error had crept in, and escaped my observation. At length, not being able to discover any, I referred to the experimental results, the greater part of which were in favour of my own theoretical deductions: still, however, as these were different beams, and many of the deflections considerable, while the investigation was supposed to apply only to those cases in which it was very small, I was still doubtful, and therefore procured three pieces of fir, each 6 feet 6 inches in length, and 2 inches in depth, by $1\frac{1}{2}$ inches in breadth, and of very uniform texture: these pieces were rested on two props, as represented in Plate V.; first, at the distance of 3 feet, and then at 6 feet.

If, therefore, the deflections varied as the square of the length, according to the results of M. Girard, the deflections ought to be, in the second case, *four times* what they were in the first; but if the deflections were as the cubes of the lengths, as they should be according to my deduction, then the deflection would be *eight times* as much. I accord-

ingly made the experiments with great care: and the following are the results that were obtained.

No. 1.

Feet long.	*Inches deep.*	*Breadth.*	*Weight, lbs.*	*Deflection.*
3	2	1½	120	·09
3	2	1½	180	·12
6	2	1½	120	·68
6	2	1½	180	1·00

The same Piece.

3	1½	2	120	·19
3	1½	2	180	·28
6	1½	2	120	1·38
6	1½	2	180	1·91

No. 2.

3	2	1½	120	·10
3	2	1½	180	·15
6	2	1½	120	·72
6	2	1½	180	1·05

The same Piece.

3	1½	2	120	·18
3	1½	2	180	·28
6	1½	2	120	1·30
6	1½	2	180	2·00

No. 3.

3	2	1½	120	·07
3	2	1½	180	·11
6	2	1½	120	·65
6	2	1½	180	·96

The same Piece.

3	1½	2	120	·16
3	1½	2	180	·24
6	1½	2	120	1·25
6	1½	2	180	1·85

95. It was impossible, after these experiments, any longer to doubt the correctness of the preceding investigations; the deflection of the 6 feet beams answering so very nearly to the cube, or to eight times that of the same at 3 feet. With regard to the deflection being inversely as the cube of the depth into the breadth, that is, inversely, as $a\,d^3 : a^3\,d$, or as $a^2 : d^2$, in the above experiments: this also is confirmed as far as the comparison can be made, but the difference in these two dimensions is not so great as in the lengths, and therefore the results, perhaps, not so conclusive.

M. Girard makes the deflections inversely, as $a\,d^2 : a^2\,d$; that is, in the above cases, as $a : d$, which by no means agrees with the above results: the discrepance will, however, be best seen by computing the deflections; first of the long beam from that of the short one being given, and comparing them with those determined from experiment; and then computing the deflections of the beams in the direction of their least depth, from those given for their greater.

	Feet.	*lbs.*	*Deflection, computed according to M. Girard.*	*Deflection from preceding Formulæ.*	*Deflection from Experiments.*
No. 1	6	120	·36	·72	·68
	6	180	·48	·96	1·00
No. 1	6	120	·76	1·52	1·38
	6	180	1·12	2·34	1·91
No. 2	6	120	·40	·80	·72
	6	180	·60	1·20	1·05
No. 2	6	120	·72	1·44	1·30
	6	180	1·12	2·24	2·00
No. 3	6	120	·28	·56	·65
	6	180	·44	·88	·96
No. 3	6	120	·64	1·28	1·25
	6	180	·94	1·92	1·81

It only requires a comparison to be made between the last column and the other two, to decide which of the two formulæ best agrees with the actual state of the beam's deflection.

96. The above are obtained from a comparison of the lengths of the beam: let us now make a similar comparison, as depending upon their depth and breadth.

	Feet.	lbs.	*Deflection according to M. Girard.* Def. $\propto \frac{1}{a\,d^2}$	*Deflection from the Formulæ.* Defl. $\propto \frac{1}{a\,d^3}$	*Deflection from Experiment.*
No. 1..	3	120	·12	·16	·19
....	3	180	·16	·21	·28
No. 1..	6	120	·91	1·21	1·38
....	6	180	1·33	1·77	1·91
No. 2..	3	120	1·33	1·77	1·80
....	3	180	·20	·27	·28
No. 2..	6	120	·96	1·28	1·30
....	6	180	1·40	1·87	2·00
No. 3..	3	120	·10	1·43	·16
....	3	180	·157	2·24	·24
No. 3..	6	120	·928	1·32	1·25
....	6	180	1·371	1·95	1·85

Here, again, the agreement between the last column and the preceding one is so near, in comparison with that computed according to M. Girard's principle, as to leave no doubt concerning the legitimacy of our formulæ.

97. Still, however, I was desirous of farther proof, and therefore procured three pieces of very clean fir, free from knots, 10 feet 6 inches long, 3 inches deep, and 1½ inch in thickness: and an ivory scale

very accurately graduated into 40ths of an inch, which was now fixed to the batten, instead of the scale of 10ths of inches hitherto employed: by which means the deflections could be accurately observed to within about $\frac{1}{80}$th of an inch.

One of the beams was now laid on with the props 9 feet apart, and put weights gradually added till the deflection was 27 of the equal parts on the scale: I then unloaded it, and set the props 6 feet asunder, and applied again the same weights, and the deflection was exactly eight divisions.

Now, in case of the deflections being as the square of the length, we ought to have had

$9^2 : 6^2 :: 27 : 12$ for the deflection at 6 feet.

But if the deflections were as the cubes,

$9^3 : 6^3 :: 27 : 8$ precisely the same as it was found to be by the experiment.

The props were then brought to the distance of 3 feet; and the same weights being used, the deflection was exactly $\frac{1}{40}$th of an inch, or one division: whereas it ought, according to M. Girard, to have been $\frac{3}{40}$ths, or three divisions.

The second batten was now laid on at 9 feet, and brought to a deflection of $40\frac{1}{2}$ divisions; the same weights brought it at 6 feet to $12\frac{1}{2}$ divisions, and at 3 feet to $1\frac{1}{2}$; whereas if the deflections had been as the squares, they ought to have been 18 and $4\frac{1}{2}$ respectively.

98. The third beam was deflected to 54 divisions at 9 feet, and the same weights brought it to $16\frac{1}{2}$ at 6 feet, and to 2 divisions at 3 feet; instead of 24 and 6, as required by the law which M. Girard had deduced from his experiments.

I next tried each of the pieces again at the distance of 6 feet, laid in the contrary way, viz. with their least thickness vertical; and placing on each the same weights as had been before employed, the deflections were, for

No. 1	32 divisions.
No. 2	48 ditto.
No. 3	64 ditto.

Which shew that the deflections were also as the cubes of the depth into the breadth, and not as the squares, as determined by M. Girard; for had that law obtained, these deflections would have been 16, 24, and 32.

98. After the preceding experiments were gone through, I made the following series on the same battens, and have computed, in every case, the value of the constant quantity, which we may call the elasticity, E, from the formula $\frac{l^3 W}{a d^3 \delta} = E$, the reduced mean of which is $E = 5317610$, whence we have $\frac{l^3 W}{a d^3 \delta} = 5317610$, from which any one of these five quantities may be found, when the other four are given.

TABLE

99. *Of the Deflections of Fir Battens.*

Weight in lbs.	Deflections at			Computed Value of $E = \frac{l^3 W}{a d^3 \delta}$			Mean Values of E.
	No. 1.—3 inches deep, 1½ inch thick. Sp. Gr. 584.						
	10 *feet.*	8 *feet.*	6 *feet.*	10 *feet.*	8 *feet.*	6 *feet.*	
70	0·65	0·31	0·16	4594960	4918816	4838512	5047599
120	1·05	0·51	0·26	4876144	5139848	5104296	
135	1·18	0·57	0·29	4881184	5227256	5148144	
150	1·30	0·64	0·31	4922960	5195848	5333328	
165	1·44	0·71	0·35	4888888	5076736	5213624	
180	1·57	0·77	0·37½	4893288	5094816	5308144	
	No. 2.—1½ inch deep, 3 inches thick. Sp. Gr. 558.						
	8 *feet.*	6 *feet.*	4 *feet.*	8 *feet.*	6 *feet.*	4 *feet.*	
35	·625	·275	·075	4893300	4691800	5097200	4877950
50	·825	·400	·112	5295800	4608000	4876200	
65	1·12	·525	·150	5071300	4564100	4723400	
80	1·36	·625	·180	5140100	4718600	4855600	
	No. 3.—3 inches deep, 1½ inch thick. Sp. Gr. 640.						
	10 *feet.*	8 *feet.*	6 *feet.*	Computed Value of $E = \frac{l^3 W}{a d^3 \delta}$: 10 *feet.*	8 *feet.*	6 *feet.*	Mean Values of E.
70	·501	·275	·114	5961400	5560600	5658900	5693427
120	·875	·467	·195	5851400	5613400	5671400	
135	1·000	·525	·220	5760000	5617400	5655300	
150	1·125	·687	·242	5688900	5582300	5712400	
165	1·237	·640	·265	5691300	5619100	5751500	
180	1·350	·700	·287	5688900	5617400	5780100	
	No. 3.—viz. the same Beam 1½ inch by 3 inches.						
	8 *feet.*	6 *feet.*	4 *feet.*	Computed Value of E, &c.: 8 *feet.*	6 *feet.*	4 *feet.*	
35	·55	·237	·70	5560600	5444100	5461300	5651466
50	·775	·327	1·00	5637500	5707100	5461300	
65	1·02	·425	1·25	5632900	5708600	5679800	
80	1·50	·512	1·50	5867000	5832000	5825400	

Mean.........$E = 5317610$.

100. As a farther confirmation of the preceding deductions, the following, from M. Dupin's experiments, may be added, which I had not seen when the above was written. The pieces on which M. Dupin's experiments were made, were 2 metres in length, and of various lateral dimensions, viz. 1, 2, and 3, &c. centimetres, to a decimeter in the squareage; they were performed with care, and conducted with great ability.

101. The following are some of the principal theorems which this author has drawn from his experiments and investigations, as connected with this part of our inquiry; viz.

1. The deflections of the same beam resting on props at each end, and loaded in the middle with small weights, are as those weights.

2. When the same piece is rested on props at the same distance, and loaded at its middle point with different small weights; these weights are reciprocally proportional to the radius of curvature at that point; and the curvature itself is consequently proportional to the weights.

3. The deflection is, *cæteris paribus*, inversely as the cube of the depth; also, the depth being the same, the deflection is inversely as the breadth.

4. The deflection is, therefore, *cæteris paribus*, directly as the cube of the length.

From which it necessarily follows, agreeably to the preceding deductions, that $\frac{l^3 W}{a d^3 \delta} =$ a constant quantity.

5. M. Dupin also demonstrates experimentally, the ratio which we have stated between the deflec-

tion of beams supported at each end, and loaded in the middle, with the deflection of the same when the weight is uniformly spread; at least his experiments give results approximating towards that ratio, viz. experimentally he has found it to be as 19 : 30, while the theory required the ratio of 5 to 8; or reducing both to the same antecedent, the first is as 95 to 150, and the second as 95 to 152, which is as nearly correct as it is possible to expect, considering, in the first place, that it is impossible practically to distribute the weights, so as to have them perfectly uniform; and in the second, that the investigation belongs only to infinitely small deflections; while experimentally they are rendered sufficiently obvious to be submitted to actual measurement. The same author has found various other interesting results; but we cannot allow any farther abstracts in this place.

102. It is important to observe, before concluding this chapter, that all the foregoing investigations have been made exclusively with reference to rectangular beams, and that they must only be considered as being applicable to that form; for, notwithstanding we have throughout made our deductions from a comparison of the depths, breadths, &c., it is obviously not the depth of the whole beam, but that of its neutral axis, on which the deflection depends; but as the latter, in rectangular beams, is always as the whole depth, we may use the one for the other indifferently, and we made choice of the latter for the sake of simplicity: the reader may, however, after reading the chapter on the situation of the neutral axis, apply all the preceding prin-

ciples to beams of every form, and in any given position.

Practical Deductions.

103. The following practical deductions flow immediately from the preceding investigations, and with them we shall conclude this chapter.

1. It has been shewn that the successive deflections are directly as the weight and cube of the length, and reciprocally, as the breadth and cube of the depth, or that the elasticity

$$\frac{W \times l^3}{a\, d^3\, \delta} = E \text{ is a constant quantity;}$$

or, which is still the same, that

$$\frac{W \times l^3}{E\, a\, d^3} = \delta.$$

2. And this formula is equally applicable to beams fixed at one end and loaded at the other, and those which are supported at both ends and loaded in the middle; but the value of E in the one case will be to that in the other, as 32 to 1. (Art. 91.)

3. And hence it follows, that in order to preserve the same stiffness in beams, the depth must be increased in the same proportion as the length, the breadth remaining constant.

4. In square beams of different lengths, the stiffness will be the same, when $s^{\frac{4}{3}}$ is as l, s being the side of the square, and l the length.

5. If the depth is given, the stiffness will be the same when a is as l^3, or when $a^{\frac{1}{3}}$ is as l.

6. The deflection of different beams arising from

their own weight, having their several dimensions proportional, will be as the square of either of their like lineal dimensions. For it has been seen that in all these cases $\frac{l^3 W}{a\, d^3\, \delta} = E$, a constant quantity: and if, therefore, we suppose each of these dimensions to be increased m times; then the weight W will be increased m^3 times, and we shall, therefore, have

$$\frac{l^3\, m^3\, W\, m^2}{a\, m \cdot d^3\, m^3\, \delta} = E, \text{ or } \frac{l^3\, W\, m^2}{a\, d^3\, \delta} = E;$$

consequently, since E is the same in both, δ must have varied as m^2.

The same will apply to beams loaded throughout proportional to the dimensions; and it is a fact which ought to be kept constantly in view in the construction of models on a small scale, of works intended to be executed on a large one.

7. With regard to the ultimate deflection of beams before their rupture, the same relations do not obtain; for it is obvious, from what has been already stated, (Art. 91,) that the depth being the same, the element of deflection will, in the breaking state of the beam, be constant; and, consequently, the ultimate deflection will in this case be as the square of the length, and it will be inversely as the depth when the length is the same; and if both these dimensions remain constant, the last deflections will be constant also, whatever may be the breadth of the beam.

The formula, therefore, applicable to this case, is $\frac{l^2}{d\, \Delta} = U$, a constant quantity, where Δ is the last deflection, l the length, and d the depth of the beam.

But little dependence, however, can be placed on this last deduction, because the law of deflections becomes very uncertain, after the elasticity has ceased to be perfect; which is some time before the rupture takes place.

We must defer drawing other conclusions from the investigations contained in this chapter, till we have examined more particularly the strain upon beams arising out of the various circumstances under which that action may take place.

On the Mechanism of the Transverse Strain of Beams, as depending on their Length, Depth, Manner of Fixing, &c. &c.

104. A BEAM of timber, as ACFI, (*plate* I. *fig.* 5,) fixed with one end in a wall, and loaded with a weight W, at the other, will be deflected from its first horizontal position AH, into the oblique direction AF; supposing it in the present instance, for the sake of simplicity, inflexible in every point, except in the section of fracture AC; and this deflection will take place, as before remarked, by means of a rotation performed about the neutral axis n.

Now the force which has a tendency to produce this rotation will obviously be the weight W, multiplied by the length of the lever n F, into the cosine of the angle n FB; and the force opposed to it will be the resistance of all the fibres in $n\,a$ to compression, *plus* the resistance of all those in n A to extension; and the sum of these two forces, in the instant before fracture, must necessarily be in equilibrio with the former, viz. with l W *cos.* n FB:

our present object, however, is not to consider the nature of the resisting forces, but of the exciting force; and this, as we have seen above, when expressed analytically, is

$$F = n\,F \times cos.\ n\,F\,B \times W, \text{ or } F = l\,W\,cos.\ \Delta$$

where F denotes the force, or strain, $n\,F = l$; the weight $= W$; and the angle $n\,F B$, of deflection, $= \Delta$.

It will be observed, that n F is not the length of the beam, but the distance of the neutral axis from the point on which the weight is suspended; nor is the angle n FB what has been at present considered as the angle of deflection; but as the depth of beams are generally small in comparison of their length, and the depth of the neutral axis still smaller, we shall in what follows, except the contrary be expressed, consider l as the length of the beam, and Δ as the angle of deflection, as it will simplify the investigation, and can produce no sensible error.

When a beam, instead of being fixed at one end into a wall, is merely rested on a support at its middle point, and loaded at each end, the tension of the upper fibre is still the same as in the former case, as before explained in treating of the deflection; the length of the beams in the latter instance being supposed double of what it is in the former; that is, supposing the beam F F′, *fig*. 8, to be double A F, *fig*. 5, then the three weights being equal, the tension of the fibre A b, in both cases, will be the same; excepting only so much of it as depends

upon the cosine of the angle of deflection, which in *fig.* 8 will be only half that in *fig.* 5: the same general expression, however, will apply in both cases, by merely changing l in the former into $\frac{1}{2}\,l$ in the latter; so that we shall have in this case

$$F = \tfrac{1}{2}\,l\,W\,cos.\ \Delta.$$

105. Now, a beam resting on a fulcrum, C, in the middle of its length, as in *fig.* 8, and acted upon by two weights W, W', has commonly been considered in the same state with regard to the strain upon it, as the equal beam F F', *fig.* 9, which is rested on the two props F F', and loaded with a double weight, P, at its centre; but it will be easy to demonstrate that this is not a correct conclusion.

In the first place, it is obvious that the resistance of the props is not made in a direction parallel to that of the vertical weight P, but perpendicular to the arms of the lever F n, F' n; and therefore, that the beam is, with regard to its strain, kept in equilibrio by the action of the three forces, F O, F' O, and O R; the former F O, F' O, being supposed perpendicular to F n, F' n.

The reaction of the fulcrums F, F', will therefore be to the weight P, as F O, to half O R, or O C; or as radius to the cosine of the angle F O n, or n' F' C; that is, as radius to the cosine of the angle of deflection.

Hence, when a beam is rested upon two fixed props, and loaded at its middle point by any weight, P, the strain upon that middle point, arising from the reaction of the props, will be found by the following proportion, as

$$OC : OF :: \tfrac{1}{2} P : \frac{FO \times P}{2\,OC}, \text{ or}$$

$$cos.\ \Delta : rad. :: \tfrac{1}{2} P : \frac{P\ rad.}{2\ cos.\ \Delta}, = \text{or} \frac{P}{2\ cos.\ \Delta},$$

taking radius equal to unity; or if we call half P = W, then, according to our former notation,

$$F = \frac{\tfrac{1}{2} l \times W}{cos.\ \Delta} = \frac{l \times P}{4\ cos.\ \Delta}.$$

This supposes the arms of the lever F n, F′ n, to remain of the same length; but it is obvious that this is also an erroneous hypothesis; for the props, or fulcrums, being fixed, these arms, either by the stretching of the fibres, or by the piece of wood slipping between the points of support, are more and more lengthened as the piece descends; viz. the length of the lever is to half the distance of the props, as *rad.* to *cos.* Δ: and, consequently, the strain on this account is again increased in the ratio of $\frac{rad.}{cos.\ \Delta}$ or $\frac{1}{cos.\ \Delta}$ to radius 1; whence, by introducing this consideration, our former expression becomes

$$F = \frac{l\,P}{4\ cos.^2\ \Delta} = \frac{l\,P\ sec.^2\ \Delta}{4}.$$

106. As I shall employ these results to explain what has hitherto been treated as an anomaly in the

experiments of Buffon and others, it may not be amiss to examine the question a little more particularly, especially as it seems to have escaped the attention of other authors.

Let, then, A C B, *(plate* II. *fig.* 1,) represent a beam of timber, or simply a lever, which, in the first place, we will suppose to be kept in equilibrio by the two equal weights W, $\acute{W}$, and the resistance of the fulcrum C, or by a weight P, acting in an opposite direction C Q; then it is obvious that the weight P must be exactly equal to the two weights W, $\acute{W}$, or $P = 2\ W$, the lever being supposed void of gravity. But the effect of the weights W, $\acute{W}$, on the two levers A C, B C, as they relate to any strain at C, may be produced by two less weights w, $\acute{w}$, acting perpendicularly to the latter; and these less weights, from the nature of the composition and resolution of forces, are to the two given weights W, $\acute{W}$, in the ratio of O B, or O A, to O C.

If, therefore, the lever A B be kept in equilibrio by the weights w, $\acute{w}$, in the directions A O, B O, the reaction of the fulcrum, that is, the weight P, must be reduced in the ratio of $O\ C^2 : O\ B^2$; for the weights themselves are less in the simple ratio of these lines, and their perpendicular action is also less in the same proportion; and, consequently, the resistance at the fulcrum, or the weight P, will be decreased in the duplicate ratio of O C to O B, or as $O\ B^2 : O\ B^2$. And, on the other hand, if the weight P remain the same in both cases, then the

equilibrium will require the weights *w*, *w*, to be increased in the ratio of $OB^2 : OC^2$; and, consequently, the effect of these on the two levers A C, B C, to produce a fracture or strain at C, will have the same increased energy.

The reader will perceive immediately that these two cases of equilibrium are similar to those of the two beams in *fig.* 8 and *fig.* 9, *plate* I., and that they agree with our former deductions;

$$\text{the first being } F = \tfrac{1}{2}\, l\, W \, cos.\, \triangle = \tfrac{1}{4}\, l\, P \, cos.\, \triangle,$$

$$\text{and the second, } F = \frac{\tfrac{1}{2}\, l\, W}{cos.\, \triangle} = \frac{l \times P}{4\, cos.\, \triangle},$$

where these two forces, or strains, are obviously to each other in the ratio of $rad.^2 : cos.^2\, \triangle$, or as the square of radius to the square of the cosine of deflection.

In this case, however, the length of the lever is not changed, because the weights are supposed to act at a fixed point; whereas in the former case, that is, when the beam is rested on two props, there is an actual lengthening of the arms of the lever; and in the latter instance, therefore, as before shewn, we must increase the strain by multiplying the latter formula by $\frac{1}{cos.\, \triangle}$, or, the strain

$$\text{in the first case} = \tfrac{1}{4}\, l\, P\, cos.\, \triangle,$$

$$\text{and in the second} = \tfrac{1}{4}\, l\, P \times \frac{1}{cos.^2\, \triangle};$$

that is, they are to each other as $cos.^3$ to $rad.^3$; whereas all writers that I am acquainted with on this subject consider them equal to each other.

Some mathematical readers may probably think I have been much more minute and explicit in the preceding investigation than was necessary; but those who are not so conversant with the resolution of forces, may not disapprove of the pains I have taken to render the deductions clear and satisfactory.

It may not, however, be improper to remark, that although the *cos.*^{2} of the angle of deflection being introduced into the general formulæ, may serve to explain some anomalies in the final results of different sets of experiments; it is a quantity which may always be dispensed with when our object is only to obtain the proper dimension of beams for building, or other practical applications; because in these cases the deflection is always very inconsiderable, and its cosine little less than radius: in all cases, therefore, except when it is in contemplation to compare the ultimate results of different experiments, we shall omit the introduction of the *cos.* Δ, and consider the straining forces under the more simple form $F = l\,W$, or $F = \frac{1}{4}\,l\,W$, according as the beam is fixed at one end, or supported at both: writing in the latter expression W, for what has been before denoted by P, viz. the suspended weight.

107. Let us now endeavour to ascertain the strain upon the centre of a beam which is loaded at that point, having each of its ends fixed in a wall, or other immovable mass.

Here it is obvious, that the whole weight is not employed in producing the strain and consequent fracture of the middle section, a part of it being

required to produce the strain and deflection at the points of fixing; consequently, beside the weight necessary to cause a fracture, or to produce any given deflection in a beam merely supported, so much additional weight must be added, when the beam is fixed at each end, as will deflect the two half-lengths to the same degree; that is, referring to *fig.* 10, *pl.* II., the weight W must be greater than would be required to deflect the supported beam, by as much as it is necessary to deflect the two half-beams. But we have seen that it will require four times the weight to produce the same deflection in a beam supported at each end, as is requisite to produce the same quantity in a beam of half the length (Art. 90); consequently, if we suppose the weight W, in the present instance, to be divided into six equal parts, four of these will be exerted in producing the deflection of the middle point, and one of each of the remaining two in producing the deflections at the points of fixing; therefore, only $\frac{2}{3}$ds of the whole weight is employed in producing the centre deflection. The strain, therefore, *on the centre of the beam*, when fixed at each end, is to the strain arising from the same weight when it is merely supported as $2:3$; and consequently the weight necessary to produce the fracture will be as 3 to 2, which accords very accurately with experiment.

Our formula, therefore, in this case, will be

$$F = \tfrac{1}{6}\, l\, W, \text{ or more accurately, } F = \tfrac{1}{6}\, l\, W, \ sec.^2\, \Delta.$$

All other theories that I am acquainted with give the ratio of 4 to 2, and the beam is supposed to be

equally liable to fracture at the ends as in the middle; but a mere inspection of the figure, with a mental reference to the actual experiment, is sufficient to shew the fallacy of such an hypothesis. In fact, in every experiment that I made, after the complete fracture in the middle, the two fragments had been so little strained at the points of fixing, that they soon after recovered their correct rectilinear form.

If the beam, instead of being fixed at each end, were merely rested on two props, and extended beyond them on each side equal to half their distance; and if weights w, w', *fig.* 11, were suspended from these latter points, each equal to one-fourth the weight W, then this would be double of that which would be necessary to produce the fracture in the common case: for, dividing the weight W into four equal parts, we may conceive two of these parts employed in producing the strain or fracture at E, and one of each of the other parts as acting in opposition to w and w', and by these means tending to produce the fractures at F and F'.

This is the case which has been erroneously confounded with the former, but the distinction between them is sufficiently obvious; because here the tension of the fibres, in the places where the strains are excited, are all equal, whereas in the former the middle one was double of each of the other two.

Parent and Belidor, in their experiments, and indeed all experimentalists except Musschenbroeck, make the strength of their beams, when fixed at the ends, to the same when merely supported, in the

ratio 3 to 2; but theorists have always made the ratio that of 4 to 2, as above stated, which, however, is obviously erroneous.

108. At present we have considered the load as being placed upon the middle of the beam; let us now endeavour to ascertain what strain will be excited in it, when the weight is placed in any other part than the centre, as at C, *fig.* 2, *plate* II.

Here, since the tension of the fibre *a b* is the same, whether we estimate it towards F, or F′, we may suppose the weight, W, to be divided into two weights, which shall have to each other the ratio of I C to I′ C; that is,

$$\text{as } I\,I' : I\,C :: W : \frac{I\,C \times W}{I\,I'},$$

$$I\,I' : I'\,C :: W : \frac{I'\,C \times W}{I\,I'}.$$

Then it is obvious, that whether we consider the first of these weights as acting at the point C of the lever C I′, or the latter as acting at the point C of the lever C I, or both of them as acting at the point C of the beam, or compound lever, I I′, the strain or tension of the fibre *a b* will be the same, and will be expressed by

$$F = \frac{I\,C \times W}{I\,I'} \times I'\,C = \frac{I\,C \times I'\,C \times W}{I\,I'};\ \text{or,}$$

$$F = \frac{I'\,C \times W}{I\,I'} \times I\,C = \frac{I'\,C \times I\,C \times W}{I\,I'}.$$

That is, if l be taken to denote the length of the beam, I I', and m and n, the two distances I C, I' C, then,

$$F = \frac{mn}{m+n} W = \frac{mn}{l} W.$$

That is, the strain varies as the rectangle of the two parts into which the beam is divided by the point of suspension: and hence it follows, that the strain will be the greatest when the rectangle is the greatest; that is, when the weight acts at the centre.

109. Let us now take the case of two weights suspended from any two points of a beam, to determine the strain upon the beam at any given point.

Conceive F I I' F', *plate* II. *fig.* 3, to be a beam resting on the two props F F', and having two weights, equal or unequal, suspended from the two points D, E; then, from the preceding formula, it appears that the strain at D is

$$F = \frac{ID \times DI'}{II'} \times W;$$ and the strain at E is

$$F = \frac{IE \times EI'}{II'} \times W'$$

Now, in order to find the strain at any other point, C, we have only to make the following proportions, viz.

$$DI' : CI' :: \frac{ID \times DI'}{II'} W : \frac{ID \times CI'}{II'} W =$$ the strain at

C, as arising from that at D; and again,

$$E\,I : I\,C :: \frac{I\,E \times E\,I'}{I\,I'}\,W' : \frac{I'\,E \times C\,I}{I\,I'}\,W' = \text{the strain at C,}$$

as arising from that at E.

Consequently, the whole strain at C, arising from both weights, will be expressed by

$$F = \frac{I\,D \times I'\,C \times W + I'\,E \times I\,C \times W'}{I\,I'}.$$

110. From this general formula may readily be deduced that for any particular case: for example,

1. Suppose the beams uniformly loaded throughout, and the stress at any point C required.

In this case, D and E will be the centre of gravity of the two parts I C, and C I′; consequently, I D $= \frac{1}{2}$ I C and I′ E $= \frac{1}{2}$ C I′; whence the expression becomes

$$F = \frac{(\frac{1}{2}\,I\,C \times I'\,C \times W) + (\frac{1}{2}\,I'\,C \times I\,C \times W')}{I\,I'}; \text{ or,}$$

$$F = \frac{I\,C \times I'\,C \times (W + W')}{2\,I\,I'}.$$

Where $(W + W')$ and I I′ being constant, it follows that F varies as the rectangle I C × I′ C; that is, in this case, the strain at any point C varies as the rectangle of the two parts into which the beam is divided by that point.

2. Suppose, again, as another example, that the weights, W, W′, are equal to each other, and that C

is the centre of the beam; then, since $I'C = IC = \frac{1}{2} II'$, and $W = W'$; the general expression becomes, in this particular case,

$$F = \frac{(ID + I'E) \times I'C \times W}{II'} = \frac{ID + I'E}{2} \times W.$$

And if we further suppose $ID = IE$, then it becomes simply

$$F = ID \times W.$$

Now, if both weights acted at the centre, it appears, from our preceding investigation, that

$$F = \frac{1}{4} II' \times (2W) = \frac{1}{2} II' \times W = IC \times W.$$

Whence the strain in the two cases will be to each other as I D to I C; and hence the following practical deduction; viz.

111. When a beam is loaded with a weight, and that weight is appended to an inflexible bar or bearing, as D E, *fig.* 4, the strain upon the beam will vary as the distance I D, or as the difference between the length of the beam and the length of the bearing; for the bearing D E being inflexible, the strains will be exerted in the points D and E, exactly in the same manner as if the bearing was removed, and half the weight hung on at each of these points. This remark may be worth the consideration of practical men in various architectural constructions.

112. In the same manner as in Art. 110, it may be shewn, that if a beam be loaded with many

weights, W, W', W'', W''', &c. as in *fig.* 5, all equal to each other, and every two of which are equally distant from the centre, the strain excited on the middle point C will be expressed by

$$F = (I\,D + I\,D' + I\,D'' + \&c.) \times W.$$

Hence, if the length of the beam be l, and the number of equal weights m, and the sum of all the weights W, then the above becomes

$$F = \left(o + \frac{l}{m} + \frac{2\,l}{m} + \frac{3\,l}{m} + \&c.\ \frac{\frac{1}{2}\,m\,l}{m}\right) \times \frac{W}{m};\ \text{or,}$$

$$F = \frac{l\,W}{m^2} \times (1 + 2 + 3 + 4,\ \&c.\ \tfrac{1}{2}\,m);\ \text{or,}$$

$$F = \frac{l\,W}{m^2} \times \frac{(\frac{1}{2}m+1)\frac{1}{2}m}{2} = \frac{\frac{1}{4}\,l\,W\,m^2 + \frac{1}{2}\,l\,W\,m}{2\,m^2} = \tfrac{1}{8}\,l\,W + \frac{l\,W}{4\,m}.$$

Hence, when the weight is uniformly distributed through the whole length, the number of points of suspension, m, becoming infinite, the last term of the preceding expression, $\frac{l\,W}{4\,m}$, vanishes; and there results

$$F = \tfrac{1}{8}\,l\,W,$$

for the strain on the centre of a beam, when the weight W is uniformly distributed throughout its length; which is half what it would be if it were all suspended from its middle point.

113. At present the weight has been supposed to act in a direction perpendicular to the fibres; that is, we have not taken into consideration the different deflections to which the beam may be exposed in consequence of the different positions of

the weight; and it has been before explained, that it is not necessary to introduce the latter datum while we are merely contemplating the comparative strengths and strains of beams for architectural and mechanical constructions, in which the deflections are always inconsiderable, but that they are essentially necessary in the comparison of experiments on the ultimate strength; and, therefore, when we treat of those comparisons, it may be necessary to modify some of the preceding results. I shall not, however, pursue the subject farther in this place, except so far as relates to the strain on beams, when the direction of the fibres and the exciting forces are placed obliquely to each other.

114. When a beam A C F I, or A' C' F' I', *fig.* 6, is placed obliquely in a wall, whether it be ascending, as in the former, or descending, as in the latter, the strain excited by the equal weights, W W', on the equal arms I C, I' C', will be the same, being in both cases expressed by

$$F = l\,W\,cos.\,I,$$

where l is the length, W the weight, and I the angle of inclination.

For, let I W, in both cases, be taken to represent the perpendicular force of the weight W, and let this be resolved into two other forces; the one, I K, perpendicular to the lever C I, and the other, K W, parallel to it; then it is obvious that K I will represent the only effective force to turn the lever about the point C; that is, the exciting force will be to

the weight W as K I : I W, or, as radius : cosine of K I W; but the angle K I W = C I L = the angle of inclination = I; therefore,

$$1 : \cos. \mathrm{I} :: \mathrm{W} : \mathrm{W} \cos. \mathrm{I} = \mathrm{I\,K},$$

which, combined with the lever C I = l, gives for the strain at C,

$$\mathrm{F} = l\,\mathrm{W} \cos. \mathrm{I}.^*$$

Therefore, while we omit the consideration of the quantity of deflection, the strain on the two beams (their lengths, weights, and inclinations being the same,) will be exactly equal to each other: and this is true, as I have before frequently observed, while we are merely considering the application of timber to architectural purposes, but fails entirely in determining the ultimate strengths.

* It was assumed by the early writers on this subject, and strangely adopted by all those who have succeeded them, that not only is the exciting force diminished in the ratio of rad. to cos., but also that the power of resistance is increased in the same ratio, viz. of cos. to rad. because they say the area of fracture C A is increased in the latter proportion; whence they conclude, that the weight necessary to break a beam in an inclined position is to the weight when it is horizontal, as $rad.^2 : cos.^2$

Nothing, however, can be more obviously false, than to suppose the power of resistance to be increased; for if the force or weight W, or W′, *fig.* 6, which is denoted by I W, be resolved into the two I K, K W; it is evident that the force I K will have the same effect upon this beam (and no other), as if the beam was placed horizontal, and loaded with a vertical weight, which should be to W as I K to I W.

There might be some plausibility for the above hypothesis in crystallised bodies, but it will certainly not apply to fibrous ones, the number of fibres on which the resistance depends being still the same.

For the deflection of the beam I C brings it nearer and nearer to a horizontal position, where the effect of the weight is the greatest; while the deflection of the descending beam I C brings it more and more towards a vertical, where the effect of the weight is the least.

Conformably to this, I have always found, that of three equal and similar beams, of which the one inclined upwards at an angle of 26°, another downwards at the same angle, and the third horizontal, that which had its inclination upwards was the weakest; the one which declined, the strongest; and the strength of the horizontal one, about a mean between both.—(See *Experiments*, Arts. 78, 81, and 82.)—It is obvious, indeed, that the ultimate strength of a beam does not depend upon its original position, but upon that which it has attained immediately before the fracture takes place.

It may be proper to observe, that in the preceding expression, $F = l \, W \cos. I$, we have only included that force which has a tendency to turn the beam about the point C: there is, however, also another exciting force, but which does not act at any mechanical advantage, that is, the force represented by K W, which in the declining position of the beam A F G I acts by tension, and in the ascending position of $\acute{A}\ \acute{F}\ \acute{G}\ \acute{I}$ by pressure: the entire expression, therefore, for the exciting force, is

$$F = l \, W \cos. I + W \sin. I.$$

But in most practical cases this latter force is very inconsiderable; first, because it does not act at any mechanical advantage through the intervention of

the lever; and, secondly, because it acts equally upon the compressed and extended fibres; and, consequently, while it increases the one of these forces, it diminishes the other, and therefore, in a certain degree, neutralises its effect on both, on which account it may in most cases be omitted: and we must necessarily omit it in this place, because its real effect depends upon the proportionality between the area of compression and that of tension, and the comparative value of these two resisting forces, the determination of which will form the subject of investigation in the following section. We shall, therefore, in this place, merely observe, that in the cases where the beam is vertical, and consequently $cos. \, I = 0$, and $sin. \, I = 1$, the former part of the expression disappears, and we have simply $F = W$; where, as in the suspended beam, W must be equal to the force of direct cohesion in the area of fracture, and in the other case it will represent the weight necessary to crush the beam with a vertical pressure.

115. At present we have only considered the strain a beam is exposed to by being charged at any point with a given weight, without making any reference to the resistance to which it is opposed. Now, this resistance obviously depends upon the figure and area of the section of the beam at the breaking point, and all theories and experiments make this resistance vary in rectangular beams as the breadth and square of the depth. That the strength or resistance is as the breadth, is obvious; because, whatever resistance any given beam offers to fracture, two, three, or more such beams will

offer two, three, or more times that resistance: and this is in fact the same as a beam of two, three, &c. times the breadth. And with regard to the depth, the resistance will be, in the first place, as the number of fibres; that is, as the depth: and, secondly, it varies as the length of the lever by which it acts; that is, as the distance of the several fibres from the centre about which the beam turns, which is also obviously as the depth; and hence, by combining the two causes, it will vary as the square of the depth when the breadth is the same: and therefore, generally, the resistance opposed to fracture by rectangular beams is as the product of the breadth and square of the depth.

If we represent the breadth of a beam of any given wood by a, its depth by d, its length by l, all in inches, and its angle of deflection by Δ, and the weight necessary to break it in lbs. by W; also, the resistance of a rod an inch square by S: then $a\,d^2\,S$ will be the resistance of the beam, whose breadth is a and depth d. Now, in the instant before breaking, there must be an equilibrium between the strain and the resistance; and hence we obtain the following equations, viz.

1. *When the beam is fixed at one end, and loaded at the other,*

$$l W \cos. \Delta = a d^2 S, \text{ or } \frac{l W \cos. \Delta}{a\, d^2} = S^*, \textit{ a constant quantity.}$$

* For the mean value of S, for different woods, see the conclusion of the following chapter.

2. *When the beam is supported at each end, and loaded in the middle,*

$$\tfrac{1}{4}\, l\, \mathrm{W}\, sec.\, ad^2\, \mathrm{S}, \text{ or } \frac{l\, \mathrm{W}\, sec.^2 \vartriangle}{4\, ad^2} = \mathrm{S},\ constant.$$

3. *When the beam is fixed at each end, and loaded in the middle,*

$$\tfrac{1}{6}\, l\, \mathrm{W}\, sec.^2 \vartriangle = ad^2\, \mathrm{S}, \text{ or } \frac{l\, \mathrm{W}\, sec.^2 \vartriangle}{6\, ad^2} = \mathrm{S},\ constant.$$

4. *When the beam in either of the two last cases is loaded at any other point than the centre,*

We shall have, in the former case, by denoting the two unequal lengths by m and n,

$$\frac{m\, n\, \mathrm{W}}{l}\, sec.^2 \vartriangle = ad^2\, \mathrm{S}, \text{ or } \frac{m\, n\, \mathrm{W}\, sec.^2 \vartriangle}{l\, ad^2} = \mathrm{S}:$$

and in the second,

$$\frac{2\, m\, n\, \mathrm{W}}{3\, l}\, sec.^2 \vartriangle = ad^2\, \mathrm{S}, \text{ or } \frac{2\, m\, n\, \mathrm{W}\, sec.^2 \vartriangle}{l\, ad^2} = \mathrm{S},$$

still the same constant quantity.

And our first formula will also apply to a beam fixed at any given angle of inclination; observing only, that the angle $\vartriangle$, in this case, will represent the angle of the beam's inclination, increased or diminished by the angle of its deflection, according as its first position is ascending or descending; or rather, it will denote the angle of the beam's inclination at the moment of fracture.

In all these cases, as we have before stated, when it is only intended to apply the results to the common application of timber to architectural and other

purposes, the angle of deflection may be omitted, and the equations then become simply,

1. $\frac{l\,W}{ad^2} = S,$ 2. $\frac{l\,W}{4\,ad^2} = S,$

3. $\frac{l\,W}{6\,ad^2} = S,$ 4. $\frac{m\,n\,W}{l\,ad^2} = S,$

5. $\frac{2\,m\,n\,W}{3\,l\,ad^2} = S.$

But in the comparison of the ultimate strength, under different circumstances, the angle of deflection must be retained; and it remains to shew how far the introduction of this datum will explain what has hitherto been considered as paradoxical in the best conducted experiments.

116. One of the most remarkable discrepancies between theory and experiment, is that which we have already explained, (Art. 107); viz. that the strength of a beam fixed at the ends is to that of a like beam merely supported, in the ratio of 3 to 2.

The next anomaly, or what has hitherto been considered as such, is that in which the strength has been observed to decrease in a higher ratio than that of the inverse of the lengths; or, which is more correct, that the strain increases in a higher ratio than the direct ratio of the lengths. Now, it appears from our formulæ, that this is what ought to be the case; for the strain being denoted by $F = \frac{1}{4}\,l\,W\,sec.\,\Delta^2$; and as the ultimate deflection, *in quantity*, varies as the square of the length, (Art. 103), the *angle* Δ will vary as the length; and con-

sequently, if the length of one beam be supposed l, and the other any number of times the same length, as $m\,l$, then the strain in the two cases will be as $\frac{1}{4}\ l\ W\ sec.^2\ \triangle$, to $\frac{1}{4}\ m\ l\ W\ sec.^2\ m\ \triangle$; and therefore, where the resistance to be overcome is the same, $\acute{W}$ will be to W as $sec.^2\ \triangle : m\ sec.^2\ m\ \triangle$, instead of being in the simple ratio of $1 : m$, as insisted upon by former writers on this subject. This defalcation of strength was observed by Buffon in his Experiments, and has ever since been considered as an inexplicable paradox. Some of the reasons assigned by Dr. Robison may probably have their effect; but it is singular that the above explanation escaped so keen a mathematician: it may not, perhaps, account for the whole discrepance in the above author's results, but it will certainly tend considerably towards reconciling them with each other. The case in which a beam is fixed at one end and loaded at the other, presents a deviation from the commonly established ratio of an opposite kind; for we have seen (Art. 115), the strain in this case is $l\ W\ cos.\ \triangle$; and since the angle $\triangle$, in this case, also varies as the length, the strain upon a beam of m times the length will be $m\ l\ W\ cos.\ m\ \triangle$; and hence, when the resistances are the same, we shall have

$$W : \acute{W} :: m\ cos.\ m\ \triangle : cos.\ \triangle,$$

instead of the simple ratio of $m : 1$; and, consequently, here the strength will not decrease so rapidly as in the inverse ratio of the lengths.

The only experiments that I am aware of bearing on this point are those of M. Parent, the results of which are published in the Academy of Sciences for

1707 and 1708, and from which the author concludes, that the weights necessary to break a beam fixed at one end and loaded at the other, and that of a beam of double the length, supported at each end and loaded in the middle, and another equal to the latter, but fixed at each end, were as the numbers 4, 6, and 10; and the preceding deductions (Art. 115) give the values of those weights as $\frac{f}{l \cos. \Delta}$, $\frac{4f}{2\,l \sec.^2 \Delta}$, $\frac{6f}{2\,l \sec.^2 \Delta}$, observing that $2\,l$ in the two latter expressions is substituted for l in the formulæ referred to, because the beams are of double length: these ratios are the same as $3\,\frac{1}{\cos. \Delta}$, $6\,\frac{1}{\sec.^2 \Delta}$, and $9\,\frac{1}{\sec.^2 \Delta}$, which, if the angle Δ be considerable, will approximate towards the above numbers: but in the reference I have seen to these experiments, neither the length of the beams nor the deflections are stated.

On the same principles, we may explain another apparent deviation of experimental results from theoretical deductions; viz. that, experimentally, the strength increases in a higher ratio than the squares of the depths, while the common theory requires them to follow exactly that ratio. But this increase of strength, or, which is the same, the decrease of strain, is apparent in our investigation; for the strain being expressed by $\frac{1}{4}\,l\,W \sec.^2 \Delta$, and the ultimate deflection being inversely as the depth, (Art. 103,) the greater this dimension is, the less will be the angle Δ, and the less, therefore, the strain; and consequently, a greater weight than that which is directly as the square of the depth will be required to produce the fracture.

On the Position of the Neutral Axis of Rotation, and the Centres of Tension and Compression.

117. We have had before occasion to remark, that when a beam is submitted to a transverse strain, being either supported at its two extremities, and loaded in the middle, or fixed at one end in a wall, and loaded at the other, it will not, as has been generally assumed, turn about its upper or lower surface, but about a line within the area of fracture; which line is, in what follows, denominated the *neutral line*, or *neutral axis of rotation.*

If the fibres of a beam (referring, for instance, to *fig.* 5, *plate* I.) were wholly incompressible, there is no doubt that the beam, when loaded at the end I, would turn about the point or line C; and every fibre of it, from C to A, would be in a state of tension.

And, on the contrary, if the fibres were wholly inextensible, then, if the beam turned at all, it must be about the point A, and every fibre from A to C would be in a state of compression.

But we know of no bodies in nature that are either inextensible or incompressible; and, therefore, the rotation of the beam will neither take place about A nor C, but on an intermediate point or line *n;* and all the fibres above that line will be in a state of tension, and those below it in a state of compression; while those which are situated so as

exactly to coincide with its plane, will be neither extended nor compressed, but be in a state perfectly neutral with regard to both.

118. It is obvious, that the fibres submitted to tension are more and more extended as they are situated further from the point *n*, and at A their extension is the greatest. The same has also place with the fibres submitted to compression, this being greatest at C; and, whatever may be the law of the forces necessary for producing these several degrees of tension and compression, or whatever may be the law of the resistances which they offer after they are produced, we may conceive some point situated between A and *n*, into which, if all the resistances to tension were united, and some point between *n* and C, into which, if all the resistances to compression were condensed, the reaction arising from these two aggregate forces would be the same as in the actual operation: and these points are what we shall designate the *centres of tension and compression;* the position of which, with that of the *neutral axis* of rotation, is now to be investigated.

119. With regard to the situation of the neutral axis, we have nothing to guide us in the determination but experiments; and these seem to indicate, that in rectangular fir beams it is at about $\frac{5}{8}$ths of the depth of the section of fracture when the beam is broken on two supports; or, at $\frac{3}{8}$ths of the same when it is broken by having one end fixed in a wall, and loaded at the other; that is, in both cases the number of fibres exposed to compression

are to those submitted to tension in about the ratio of 5 to 3.*

It is not, however, on the experiments referred to below only that I venture to state the above proportion; for it was, perhaps, pointed out more unequivocally in the usual experiments, the beams in most cases shewing very distinctly, after the fracture, what part of the section had been compressed, and what had experienced tension; the compressed fibres being always broken very short, having been first crippled by the pressure to which they had been exposed, while the lower part was drawn out in long fibres frequently 5 or 6 inches in length. A fracture of this kind is shewn in *fig*. 3, *plate* III., being a very accurate delineation of the fragments of No. 2, Art. 71.

Another criterion was found in the external appearance of the pressure before the fracture took place, which always exhibited itself in a wedge-like form, the lower point of which, when the beam was broken on two props, was commonly found to divide the depth in about the ratio above stated.

120. From all these circumstances, I conceive we may, as a first approximation, assume the neutral line in rectangular fir beams to be at $\frac{5}{8}$ths of the depth when they are broken on two supports; and at $\frac{3}{8}$ths, when they are fixed at one end in a wall.

The mechanical operation of fracture in the latter case may therefore be considered under the form

* See *Experiments* Art. 74, which were made with a view to this determination.

shewn in *fig.* 7, *plate* II, where n is the neutral axis, t the centre of tension, and c the centre of compression; w a weight equal to the tension of all the fibres in A n, and w' a weight equal to all the resistances to compression in n C: which weights and distances, or levers, $n\,t$, $n\,c$, must be such that $w \times n\,t = w' \times n\,c$; for it is this equality which determines the position of the point n. And the sum of these, when the weight W is just sufficient to produce the fracture, is equal to $W \times n\,N$; that is, when the three forces are in equilibrio, we must have

$$w \times n\,t + w' \times n\,c = W \times n\,N = 2\,w \times n\,t.^*$$

The weight w obviously depends upon the force of direct cohesion of the fibres (and may be expressed by $w = f\,a$, where f is the force of direct cohesion on a square inch, and a the area of tension), and the

* I ought to observe, that the principle here assumed has been objected to by Mr. Hodgekinson, in a highly ingenious paper in Vol. IV. of the Manchester Memoirs. This author conceives, that the simple resistance to extension and compression are the quantities which ought to be made equal to each other; whereas, I have assumed, that the areas into the depth of the centres of compression and tension, and into those forces respectively, are equal; or, which is nearly the same, I consider the wedge-like forms included in these two parts into these respective forces, as the sum of all the resistances, to be equal; and he, merely their areas. However this may be decided, it fortunately happens that the doubt applies to a part of our inquiry which is one rather of curiosity than utility; I have therefore left this section in the form in which it stood in the two former editions of this work, and I believe it will be found correct as applied to the ultimate state of the beam before fracture.

centre t upon the law of tension. If we adopt the hypothesis of Galileo, viz. that the resistances are the same through the whole section n A, then the point t will be the centre of gravity of that section: and if we suppose, with Leibnitz, that the resistances are as the tensions, then the point t will be distant from n by a quantity equal to the product of the distances of the centres of gravity and oscillation of the section, divided by its depth n A (Art. 20): that is, in the first case, $n\,t$ will be equal to half n A, and in the other, equal to one third; while the weight $w = a\,f$, which depends wholly on the force of direct cohesion, the area being constant, will remain the same in both cases; and, consequently, the results on the two hypotheses will differ from each other in the ratio of $\frac{1}{2}$ to $\frac{1}{3}$, or as 3 to 2.

Let us, then, without assuming either the one or other, examine from the results of our experiments, which of the two best agrees with the actual operation, or whether there may not be some different law which better fulfils the conditions of the question.

121. We shall select for this comparison Experiments 3 and 4, Art. 71; from the mean of which it appears that a beam, 48 inches long and 2 inches square, and supported on props at its two ends, required 1116 lbs. to produce the fracture; and, therefore, the same beams, broken at 24 inches, with one end in a wall, would have required half that weight, viz. 558 lbs.; under which latter form it will be more convenient to consider them. These

beams are chosen in preference to any other: first, because they approach very nearly to the specific gravity 600, which has been made the standard; and, secondly, that triangular beams were formed out of the fragments of them, which, as well as the beams themselves, gave very uniform results; and, thirdly, because it was from the ends of the same that the vertical pieces were made that gave the most uniform determinations relative to the force of direct cohesion, viz. the first six Experiments, Art. 58; from which it appears, that the force of direct cohesion on a square inch of fir is equal to very nearly 13,000 lbs.: to which we may also add, that the beams being short in comparison with their depth, their deflection is inconsiderable, and may therefore be omitted in the investigation.

122. The data, then, on which we shall proceed, are as follow:

1st. That a beam, 24 inches long and 2 inches square, fixed at one end in a wall, requires a weight of 558 lbs. to produce the fracture.

2d. The neutral point, or axis, is at about $\frac{3}{8}$ths of the depth of the beam.

3d. The force of direct cohesion on a square inch of the same wood is equal to 13,000 lbs.

These being given, it is required to find the situation of the centre of tension, and thence to infer the law of reaction of that force.

123. Let A F I C, (*fig.* 5, *plate* I.,) represent the beam in question, in which n A $= \frac{3}{8}$ths of the whole depth $= \frac{3}{4}$ inch; and, therefore, $\frac{3}{4} \times \frac{2}{1} = \frac{3}{2} = a$ inches, is the area of tension.

Consequently, $\frac{3}{2} \times 13000 = 19500$ lbs. is the force of tension on that area; that is, referring to our preceding formula (Art. 120) $a f = w = 195000$; the weight W is also given $= 558$; and $l = 24$: calling, therefore, $n t = x$, we have

$$2 w x = l \mathrm{W}, \text{ or}$$

$$2 \times 19500 x = 24 \times 558:$$

$$\text{Whence, } x = \frac{24 \times 558}{39000} = \frac{13392}{39000} = \frac{1674}{4875};$$

that is, $n t : n \mathrm{A} :: \frac{1674}{4875} : \frac{3}{4}$, or as 744 : 1625. This ratio, changed into a series of converging fractions, gives

$$^{*}\frac{1}{2}, \frac{5}{11}, \frac{11}{24}, \frac{38}{83}, \frac{353}{771}, \frac{744}{1625}.$$

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* The operation at length is as follows :—

```
744)1625(2
    1488
    ----
     137)744(5
         685
         ---
          59)137(2
             118
             ---
              19)59(3
                 57
                 --
                  2)19(9
                    18
                    --
                     1)2(2
                       2
                       -
                       0
```

Quotients, 2, 5, 2, 3, 9, 2

Converging Fractions, $\frac{1}{2}, \frac{5}{11}, \frac{11}{24}, \frac{38}{83}, \frac{353}{771}, \frac{744}{1625}$

See BARLOW's *Theory of Numbers.*

The first is a half, and the succeeding ones differ but a little from it: and hence, it appears, we are justified in assuming, as a first approximation, that $n\,t = \frac{1}{2}\,n\,A$. This is greatly in favour of the Galilean hypothesis, which makes the centre of tension coincident with the centre of gravity.

We have seen that, according to the supposition of Leibnitz, the distance $n\,t$ ought to have been $\frac{1}{3}$d of $n\,A$; and, therefore, that fraction should have been found in our series of converging fractions, which, however, it is not, nor any that approximates towards it.

124. The triangular beam offers a still better means of deciding between the two hypotheses; for, in the one case, that is, according to Galileo, the centre of tension, when the edge of the beam is submitted to that force, is at $\frac{1}{3}$d of the perpendicular from the base, being the same as the centre of gravity; whereas Leibnitz makes it $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$. (Art. 20.)

Let us, then, go through a similar mode of induction, as depending upon the results of the experiments on triangular beams; first, with the edge submitted to the force of tension; and, secondly, with the base exposed to the same strain.

We have before remarked, that Experiments 3, 4, 7, and 8, Art. 76, were made on the fragments of the same battens which have furnished the preceding results; and of these, Nos. 3 and 4, having been broken on props, with their edge downwards, are those we shall first examine.

These were each 24 inches long, and the side of the equilateral triangle which formed their end was

2 inches; and, consequently, the perpendicular depth = $\surd 3 = 1{\cdot}732$ inches; the weight in both experiments was 740 lbs.: and this, to proceed in the same manner as before, is equivalent to a beam of like lateral dimensions, 12 inches long, broken with one end in a wall, with its edge upwards, by a weight of 370 lbs.

The distinction between that part of the section of fracture which was submitted to compression, and that acted on by tension, was equally well marked in these experiments as in the former, *fig.* 1, *plate* III., being a very correct representation of one of the pieces: and from a measurement of it, it was found that the depth of the compression F n C m was, as nearly as possible, ·75 of an inch.

Let A B C, *fig.* 8, *plate* II., represent the section of fracture; we have A B = 2, n D = ·75, C D = 1·732, and, consequently, n C = ·982; the area of the triangle A B C = 1·732, and the area of tension N C N = a = ·55673.

Consequently, $a\,f = w = {\cdot}55673 \times 13000 \doteq$ 7237 lbs.

Also, l = 12 inches, and W = 370.

Hence, denoting the centre of tension by t, and calling $n\ t = x$; we have from the preceding formula; viz.

$$2\,w\,x = W\,l,$$

$$3 \times 7237\,x = 12 \times 370;$$

$$\text{or, } x = \frac{2220}{7237}.$$

And, since n C = ·982, or $\frac{982}{1000}$, we have

$$n\,C : n\,t :: \frac{982}{1000} : \frac{2220}{7237} :: 7106734 : 2220000.$$

The leading converging fractions approximating towards this ratio, are,

$$\frac{1}{3}, \frac{4}{13}, \frac{5}{16}, \text{\&c.}$$

The first of which being $\frac{1}{3}$d, still agrees with the Galilean hypothesis, the centre of tension and centre of gravity being nearly in the same point.

125. Again, from the result of four experiments made on triangular pieces broken on two props, with the base downwards, the neutral line, which was very distinctly marked, (particularly in that represented in *fig.* 2, *plate* III.,) was found to be at ·45 of an inch, on the side of the triangle, from the base; and both experiments 7 and 8, Art. 76, which, as before stated, were part of the same piece as the preceding, and the same length, 24 inches, were broken with 637 lbs., or reduced to specific gravity 600, 626 lbs., which is equivalent to a batten of 12 inches, broken with one end in a wall, with its base upwards, by a weight of 313 lbs., the section of which is represented in *fig.* 9, *plate* II. Now, the side A C, or B C, being 2 inches, and A N = ·45 inch, we have N C = 1·55 inch: also the area being 1·732, we find,

$$2^2 : 1{\cdot}55^2 :: 1{\cdot}732 : 1{\cdot}0402 = \text{area N C N.}$$

Consequently, $1.732 - 1.0402 = {\cdot}6918 =$ area of tension $= a$.

And again, $2 : 1{\cdot}732 :: {\cdot}45 : 3893 = \text{D}\,n$.

Hence, $\cdot6918 \times 13000 = 8993$ lbs., the direct cohesion on the area of tension, and which, as before, we shall denote by w: also $l = 12$, and $\text{W} = 313$.

Whence, calling the distance $n\,t = x$, our preceding formula, $2\,w\,x = l\,W$, gives

$$17986\,x = 12 \times 313, \text{ or}$$

$$x = \frac{626}{2997} = \cdot 208 = n\,t.$$

Now the distance of the centre of gravity of the area of tension A N N B from n is

$$n\,t = \frac{(2\,p + d)\,(p - d)}{3\,(p + d)}.\text{*}$$

Where p = C D = 1·732, and $d = n$ C = C D − D n = 1·3428, which substituted for p and d, give

$$\frac{(2\,p + d)\,(p - d)}{3\,(p + d)} = \cdot 203,$$

---

* The general expression for the centre of gravity of a surface, is

$$\text{S G} = \frac{\int y\,x\,\dot{x}}{\int y\,\dot{x}}$$

If, therefore, in the present instance, we call C D $= p$, and n C $= d$, and any variable distance $n\,t = x$, also A B $= b$, we have

$$p : b :: d + x : \frac{b\,(d + x)}{p} = y.$$

Whence the above becomes

$$\frac{\int b\,(d + x)\,x\,\dot{x}}{\int b\,(d + x)\,\dot{x}} = \frac{\int (d\,x\,\dot{x} + x^2\,\dot{x})}{\int (d\,\dot{x} + x\,\dot{x})} =$$

$$\frac{\frac{1}{2}\,d\,x^2 + \frac{1}{3}\,x^3}{d\,x + \frac{1}{2}\,x^2} = \frac{3\,d\,x + 2\,x^2}{6\,d + 3\,x} =$$

$$\frac{(3\,d + 2\,x)\,x}{6\,d + 3\,x};$$

which, when $x = p - d$, $= n$ D, becomes

$$\frac{(2\,p + d)\,(p - d)}{3\,(p + d)}$$

as above.

the distance of the centre of gravity; and we have found $n\,t$ the distance of the centre of tension $= 208$; differing from each other only by ·005, or $\frac{1}{200}$ of an inch.

126. In the preceding investigation we have gone over exactly the same process as was pursued in the first examination of this question; because we conceived it would be more satisfactory to the reader to see in what manner the conclusion was obtained, than it would be merely to state it in the form of an hypothesis, and to shew the coincidence between the theoretical and practical results; although the latter might have been more elegant and concise.

127. There can, I think, be no doubt, after the preceding deductions, however difficult it may be to explain the circumstance on physical principles, that the centre of gravity and the centre of tension are in the same, or nearly in the same, point; which is what would be the case if the tension of the fibres were the same for every degree of extension, and totally independent of the quantity of it, as first assumed by Galileo. I shall, in this place, merely offer one suggestion by way of explanation, which is, that the hypothesis "*ut tensio sic vis,*" is founded on a supposition of perfect elasticity, and admits of no limit or termination of this law: but it is obvious, that in all the bodies we are acquainted with, and particularly in timber, it can only obtain to a certain extent, after which an increase of force may be, and obviously is, accompanied by a decrease of reaction: thus referring to *fig.* 5, *plate* I., when the

fibre b A is so extended that its reaction is at its maximum, all the other fibres between b A and n fall short of that maximum value: but as more weight is applied at F, the other fibres situated below b A arrive at their maximum, while the reaction of that fibre is diminished: and as the beam is farther excited, other lower fibres come successively to the maximum of reaction, while those that have already arrived at that state are undergoing a diminution. The law of reaction till the fibres have attained their maximum, we may suppose to coincide with the state of perfect elasticity, that is, that the reaction is as the exciting force: but what may be the law by which that power diminishes, when excited beyond that which produces the maximum, I shall not attempt to assume: it is sufficient for my present purpose to establish, on general principles, that such a decrease does obtain; and, as a consequence of this, that the fracture of the beam does not take place immediately after the extreme fibre has passed its maximum, but that this event depends upon the sum of the reactions of all the fibres having arrived at, and passed, that state.

This being admitted, we see the reason why the deflections during part of the operation (viz. while the reaction of the fibres are all increasing,) are as the exciting forces; and why, after that point, they proceed more rapidly than in the proportion of the weights; and also, why these deflections are greater and greater as the beams are more and more excited.

It follows, also, as a necessary consequence of the above reasoning, that the centre of tension, at the

time of fracture, cannot be situated in the point assigned to it by Leibnitz, because this depends upon what we have shewn to be an erroneous hypothesis; viz. that the fracture happens immediately after the extreme fibre has passed its maximum of reaction.

As to the exact determination of the situation of this centre, it obviously depends upon the law of decrease of energy in the fibres after passing their maximum value, as combined with the increasing effect of the other fibres before they arrive at it: but whatever the former law may be, it will evidently have a tendency to place that centre farther from the axis, than accords with the hypothesis of Leibnitz, and, consequently, to approximate it towards that assumed by Galileo. The results of our investigations point towards the latter; and though we have not found a perfect coincidence with either, yet it appears to be so near to that of Galileo, that no sensible error will take place by supposing the centre of gravity and that of tension to be in one and the same point. It must be observed, however, that the coincidence which we have thus found between the result of Galileo's hypothesis and the above deductions, is by no means a proof of the correctness of the former: but, on the contrary, admitting the above explanation, it shews that coincidence to be merely accidental: for, instead of supposing the fibre of greatest reaction, in the breaking state of the beam, to be situated in or near the centre of the area of fracture, and the fibres both above and below this to have diminished action, on the one side in consequence of not being sufficiently excited, and on the other

from that excitation being carried too far, he supposed them all to have an equal action; which, though wholly erroneous as an hypothesis, may give very nearly or exactly the true result.

There is also another important distinction between the Galilean hypothesis and our deduction; viz. that he supposed the whole number of fibres to exert this kind of resistance: whereas it appears, from the several experiments above referred to, as well as from the nature of every material substance, that a part of them only are thus employed; and, in timber in particular, that part is comparatively but small.

This is indeed an important difference: it is on this account that he makes his triangular beams strongest with the base downwards, when supported on two props, or, when fixed the contrary way, in a wall; whereas our experiments shew the fact to be exactly the reverse. According to this author, the strength of triangular beams with the base downwards, is to that of similar beams with the edge downwards, as 2 to 1; while Leibnitz and others, who thought they were improving upon Galileo's theory, make the strength in the two cases as 3 to 1, which is still more erroneous; the strength, as derived from experiment, being at a mean in about the ratio 6 to 7.

128. To return, after this digression, to our previous object, it will be observed that we have not found a complete coincidence between the centres of tension and of gravity: but it is also to be remarked, that all the preceding calculations have been made from the position of the neutral axis, as drawn from

observation only; and, in this respect, it will readily be admitted that the utmost precision cannot be expected. The most certain index to be found in this respect is the *crushed* part, which shews itself on the side of the beam; and this was measured with care, and with as much accuracy as could be obtained with a pair of compasses, and a scale of 40ths of inches, which indeed was sufficient for the intended purpose, this being merely to find some indication of the position of the centre of tension.

In triangular beams, if we attempt to compute the position of this point, it will be found to vary very considerably, according as we assume different laws of tension: if the latter be supposed uniform, the centre of tension will be the same as the centre of gravity, and will be at $\frac{1}{3}$d of the perpendicular from the base, as we have found it to be very nearly. But supposing the force to be as the quantity of tension, according to the theory of Leibnitz, then that centre would be only at $\frac{1}{6}$th of the perpendicular; and if the force varied as any higher power of the distance from the neutral axis, it would bring that point still nearer to the base.

Whence it follows, that although we may not have assumed the situation of the neutral axis to the greatest accuracy, which was not indeed aimed at, it is sufficiently exact for deciding between the two suppositions of Galileo and Leibnitz; and a strong inference may be drawn from our calculation in favour of the former. We had in the triangular beam to decide between the fractions $\frac{1}{3}$ and $\frac{1}{6}$, and in the rectangular one between the fractions $\frac{1}{2}$ and $\frac{1}{3}$;

and the results are so very near the two former of each, as to leave no doubt as to which assumption may be most safely made.

129. We shall now therefore revert the order of our calculus, and by assuming the centre of tension and centre of gravity as coinciding, endeavour to compute the exact situation of the neutral axis, and hence to determine the position of the centre of compression, which is equally necessary to be known, in order to establish a correct theory of the strength and stress of beams.

For this purpose, it will only be requisite to have recourse to our former theorem; viz.

$$2 f a \times x = l \times W;$$

where f the force of direct cohesion, l the length of the beam, and W the breaking weight, are known quantities; and a the area of tension, and x the distance of the centre of gravity or tension, are to be determined.

Referring, therefore, to the rectangular beam which furnished the preceding results, we have $l = 24$, $W = 588$, and $f = 13000$. Whence, putting y for the depth of the area of tension, $2\,y$ will be the area, and $\frac{1}{2}\,y = x$ the distance of the centre of tension.

Therefore, $2 f a = l\,W$

becomes $2\,y^2 = \dfrac{24 \times 588}{13000}$, or

$$y = \sqrt{\frac{13392}{26000}} = \cdot 717.$$

And our measurement led us to assume this depth ·75; viz. ·033 of an inch in excess: but I have

before observed, that the utmost accuracy in this respect was not attempted: indeed, the nature of the surface to be measured precluded the possibility of determining the depth within the 20th of an inch.

130. Let us next take the triangular beam, fixed in a wall with its edge upwards, and of which the section is represented in *fig.* 8, *plate* II.

Here we have $l = 12$, $W = 370$, $A\ B = 2$, $D\ C = \sqrt{3} = 1{\cdot}732$, and the area $A\ B\ C = 1{\cdot}732$.

Make $n\ C = y$, then $n\ t = \frac{1}{3}\ y$, and by proportion

$$D\ C^{2} : n\ C^{2} :: \text{area } A\ B\ C : \text{area } N\ C\ N, \text{ or}$$

$$3 : y^{2} :: \sqrt{3} : \frac{y^{2}}{\sqrt{3}} \text{ area of tension.}$$

Whence our formula,

$$2\ w\ x = l\ W$$

$$\text{becomes } 2 \times 13000 = \frac{y\ 3}{3\sqrt{3}} = 12 \times 370, \text{ or}$$

$$y = \sqrt[3]{\frac{370 \times 12 \times 3\sqrt{3}}{2 \times 13000}} = \sqrt[3]{\cdot 887315} = \cdot 9609.$$

Which, from the measurement of the fracture, we found to be ·982; viz. ·0211 of an inch in excess.

131. Again, in the opposite position of the beam, we have $W = 313$, $l = 12$, $f = 13000$; the other dimensions being the same as before,

Let then $N\ C = y$, (*fig.* 9, *plate* II.,) then

$$3 : y^{2} :: \sqrt{3} : \frac{y^{2}}{\sqrt{3}} = \text{area } N\ C\ N.$$

Consequently, $\sqrt{3} - \frac{y^{2}}{\sqrt{3}} =$ area A N N B, that is, the area of tension.

Also from note, art. 125, the distance

$$n\,t = \frac{(2\,p + y)\,(p - y)}{3\,(p + y)}.$$

Whence the formula

$$2\,f\,a \times x = l\,W$$

becomes, (by substituting throughout p for $\sqrt{3}$),

$$2 \times 13000\left(p - \frac{y^2}{p}\right) \times \frac{(2\,p + y)\,(p - y)}{3\,(p + y)} = 12 \times 313,$$

which reduces to

$$(2\,p + y)\,(p - y)^2 = \frac{5634\;p}{13000}.$$

Or re-establishing the value of p,

$$y^3 - 9\,y = -\,{\cdot}964138.$$

From the solution of which we find $y = 1{\cdot}3365$, and consequently $1{\cdot}732 - 1{\cdot}3365 = {\cdot}3955$ of an inch; which, from observation of the fracture, was supposed to be ·3893, leaving an error of only ·0062 of an inch, a quantity much too small to be detected with our means of measuring.

132. Let us now, therefore, from our previous determinations, endeavour to discover the law which the reaction of the fibres observe in their compression.

It has already been assumed, that, whatever may be that law, the entire force of compression is equal to that of tension, and that it is this equality which determines the situation of the neutral axis. If, therefore, we knew the direct resistance to compression on any given area, as we do that of cohesion, we might proceed to find the centre of compression in a similar manner to that which led to

the centre of tension; but for want of this datum we must pursue a different method.

133. For this purpose I shall assume that a compressed particle, or fibre, placed at the same distance below the neutral axis as an extended one above it, will offer a resistance, bearing some unknown, but constant, ratio to the resistance of the extended fibre: and farther, that in the ultimate fracture the most extended fibre is always stretched to the same degree or quantity, whatever may be the depth of the beam.

It must also be admitted, that the resistance of the several fibres to compression are either uniform, or that they follow some law depending upon the quantity of compression; or, which is the same, on their distance from the neutral axis: and consequently, that there is some point in the compressed area, into which, if all the forces were united, their resistance would be the same as in the actual operation.

On these assumptions, let us take the same three experiments as have furnished the preceding results; the particulars of which, when collected, are as follow:

1. *Rectangular beam;* length 24 inches, depth 2 inches, breadth 2 inches, breaking weight 588 lbs.

Area of tension 1·434, depth of the same ·717, centre of tension ·3585, area of compression 2·566, depth of the same 1·283.

2. *Triangular beam, edge upwards;* length 12 inches, side of equilateral triangle 2 inches, breaking weight 370 lbs.

Area of tension ·5331, depth of tension ·9609,

centre of tension ·3203 ; area of compression 1·1989, and depth of the same ·7711.

3. *Triangular beam, edge downwards;* length 12 inches, side of equilateral triangle 2 inches, breaking weight 313 lbs.

Area of tension ·702, depth of the same ·3955, centre of tension ·206; area of compression 1·030, and depth of the same 1·3365.

134. Now, according to the preceding hypothesis, the compressed area, into the distance of the centre of compression; is to the area of tension, into the distance of the centre of tension in a constant ratio, whatever may be the form or magnitude of the beam.

Hence, denoting by x, y, and z, the distance of the centres of compression from the neutral line, in each of the above cases respectively, and $1 : m$ the ratio sought, we have

$$1{\cdot}434 \times {\cdot}3585 : 2{\cdot}566 \times x :: 1 : m$$
$${\cdot}5331 \times {\cdot}3203 : 1{\cdot}1986 \times y :: 1 : m$$
$${\cdot}702 \times {\cdot}206 : 1{\cdot}030 \times z :: 1 : m.$$

$$\text{Whence } x = \frac{{\cdot}514089}{2{\cdot}566} m = {\cdot}2003\, m$$
$$y = \frac{{\cdot}17075193}{1{\cdot}1989} m = {\cdot}1424\, m$$
$$z = \frac{{\cdot}144682}{1{\cdot}030} m = {\cdot}1404\, m.$$

We have thus the relative values of these three distances, and it remains for us to assume such a law of compression as shall best agree with the above relative values.

Supposing then, first, the resistance in this case

to follow the same law as in the preceding one of the tensions, we shall have the distances of the centres of gravity as follow; viz.

for the rectangular beam $\frac{1\cdot283}{2} = \cdot6415$,

the 1st triangular beam $\frac{(2\,p + d)\,(p - d)}{3\,(p + d)} = \cdot422$,

the 2d triangular beam $\frac{1\cdot3365}{3} = \cdot4455$.

And these numbers are found to be nearly in the same ratio with regard to each other as the preceding ones; for in the

rectangular beam	·2003 : ·6415 : : 1 : 3·20,
1st triangular beam	·1424 : ·422 : : 1 : 2·96,
2d triangular beam	·1404 : ·4455 : : 1 : 3·18.

The first and last of these ratios approach very near to equality; and although there is some discrepance in the second, yet the approximation is so near, as to render it unnecessary to look for any other law of compression than the one we have supposed; viz. a uniformity of resistance, independent of the quantity of compression.

135. Upon the whole, therefore, I shall venture to give the following theorems, relative to the situation of the neutral line in fir beams of any magnitude and form: viz.

1. The centre of compression and the centre of tension coincide with the centres of gravity of their respective areas.

2. The area of tension into the distance of the centre of tension from the neutral line, is, to the area of compression into the distance of the centre

of compression from the same line, in a constant ratio; which ratio approaches towards that of 1 : 3·11, (the mean of the three preceding ones;) but the exact determination of it is reserved for a subsequent investigation.

136. The above deductions, both as they relate to the law of tension and compression, I am aware, differ very essentially from what has been assumed by many eminent mathematicians; and, if theirs had not been *merely assumption*, while the above are legitimately drawn from the results of numerous experiments, I should have hesitated to promulgate them; but, resting, as they do, on the basis of experiment, (the only secure ground to build physical science upon,) they must be admitted by the mathematician; and, as to the philosopher, he will, after the example of our illustrious Newton, endeavour rather to adjust his hypothesis to the experiments, than the latter to the former.

137. I have, in a preceding article, endeavoured to account for the coincidence between the centres of tension and gravity; and it may not be amiss to offer a few remarks, in explanation of our second coincidence, viz. between the centres of compression and gravity, particularly as this appears to be much more obvious than the former. I have observed, in several parts of this work, that in nearly all our experiments, the distinction between the areas of compression and tension was decidedly marked, and that, even before the fracture, the compressed fibres were rendered very conspicuous, by exhibiting a *crushed* part, in the form of an isoceles triangle,

having its vertex in the neutral axis. Now, this being the case, and assimilating the reactions of the fibres to that of a series of spiral springs, increasing in length from the neutral axis upwards, it is obvious that the length of the spring is every where as the compressing force; and, consequently, the resistance from the vertex to the base will be the same in all points: consequently, the centre of compression ought to be the same as the centre of gravity, as we have found it.

How far the reasoning offered here, and in Art. 127, may be considered satisfactory, is not for me to determine; but as my object in this work is not to examine the question physically, but mathematically, I shall offer no farther remarks on this head, leaving a better explanation, if it can be found, to those who are more conversant with inquiries of this kind.

138. The three ratios which we found above; viz. 1 : 3·20, 1 : 2·96, and 1 : 3·18, of which the mean is, 1 : 3·11, are certainly not so nearly equal as could be wished, although they approach as near to equality as might be expected, considering the irregularities to which experiments of this kind are subject: the mean ratio, however, above found, may require some modification, and which we have the means of arriving at by a comparison of that ratio in all our principal experiments. Thus, by assuming as a general principle, that the centre of compression and centre of tension coincide with the centres of gravity of their respective areas, and knowing the breaking weight for each, the ratio in every mean experiment may thence be found;

and hence, again, a mean ratio for the general theorem.

The formula for the ultimate strength of beams, supported at each end and loaded in the middle, is

$$8 f a x = l \text{W},$$

(that is, 4 times $2 f a x$, Art. 129), where f is the force of direct cohesion, a the area of tension, and x the distance of the centre of tension from the neutral axis; or, by writing δ for the depth of tension, then in rectangular beams $x = \frac{1}{2} \delta$, also denoting the breadth by b, the depth by d, and the length by l, the above becomes

$$4 f \delta^2 b = l \text{W}.$$

Again, the depth of compression will be $(d - \delta)$, and the distance of its centre of gravity $\frac{1}{2} (d - \delta)$; whence, using $1 : m$ to denote the ratio sought, we have the following equations for finding δ and m; viz.

$$\left.\begin{array}{l} 4 f \delta^2 b = l \text{W}, \text{ and} \\ 4 f \delta^2 b : 4 f b (d - \delta)^2 :: 1 : m \end{array}\right\} \text{or,}$$

$$m \delta^2 = (d - \delta)^2, \text{ and}$$

$$\delta = \sqrt{\frac{l \text{W}}{4 f b}}.$$

From which last equation, δ becomes known, and the preceding one gives

$$m = \frac{(d - \delta)^2}{\delta^2}$$

Let us, therefore, by means of this formula, compute the values of m, corresponding to the mean results of our several experiments, and thence deduce its general mean value.

TABLE of Results for Computing the Values of δ and m.

Given Quantities.						*Computed Results.*	
Art.	*l.*	*b.*	*d.*	*f.*	*W.*	δ.	*m.*
67	15	1	1	13000	439	·3558	3·28
67	18	1	1	13000	342	·3441	3·63
68	24	1	1	13000	288	·3645	3·04
68	30	1	1	13000	237	·3697	2·91
68	36	1	1	13000	196	·3683	2·94
*70	24	1	2	13000	1119	·7186	3·17
70	30	1	2	13000	900	·7205	3·15
70	36	1	2	13000	745	·7182	3·18
71	44	2	2	13000	1255	·7283	3·04
71	48	2	2	13000	1116	·7177	3·19
72	72	2	2	13000	744	·7319	3·00
							11)34·53
							Mean value of $m = 3·14$

139. Hence, assuming this mean value of m, it follows, that in fir rods, of which the strength of direct cohesion is 13,000 lbs., the position of the neutral axis is such, that the area of tension into the distance of the centre of tension, is to the area of compression into the distance of the centre of compression as 1 : 3·14.

But, as the ratio 1 : 3·14 is rather inconvenient in many numerical computations; and as it is obvious that this ratio, the ultimate strength of the beam being supposed the same, will decrease as the force

* The experiments, Art. 69, and those on the 5 feet beams, Art. 72, are omitted in the above table; the former being wood of a peculiar quality, and the latter being noted in the experiments as very irregular.

of direct cohesion decreases, we may, without any sensible error, convert the ratio of 1 : 3·14 into that of 1 : 3, by making a corresponding reduction in the force of direct cohesion.

140. For this purpose, let us take a rod, an inch square, and calling the depth of tension x; and consequently the depth of compression $1 - x$, we have, for finding x,

$$\tfrac{1}{2}x^2 : \tfrac{1}{2}(1-x)^2 :: 1 : 3{\cdot}14; \text{ or,}$$
$$3{\cdot}14\,x^2 = 1 - 2x + x^2; \text{ or,}$$
$$x^2 + \frac{2}{2{\cdot}14}x = \frac{1}{2{\cdot}14}.$$
$$\text{Whence, } x = \frac{1}{2{\cdot}14} \times (\sqrt{3{\cdot}14} - 1) = {\cdot}3607.$$

But if we assume the ratio of 1 : 3, and denote the depth of tension by y, and that of compression by $1 - y$, we have,

$$\tfrac{1}{2}y^2 : \tfrac{1}{2}(1-y)^2 :: 1 : 3:$$
$$\text{whence, } y = \tfrac{1}{2} \times (\sqrt{3} - 1) = {\cdot}3660.$$

And since the ultimate resistance is as the force of direct cohesion into the square of the depth of tension, and the former being supposed the same in both cases, we have,

$$(3660)^2 : (3607)^2 :: 13{,}000 : 12{,}630, \textit{ nearly}$$

the reduced strength of direct cohesion sought.

141. This modification of our preceding theorem, viz. changing the strength of direct cohesion from 13000 to 12630; and, as depending upon this, the ratio 1 : 3·14 into that of 1 : 3, will make no difference in any computed results relating to rec-

tangular beams, and very little in those of other forms; and as it will, at the same time, throw great facility into our numerical calculations, we may, in any subsequent investigations, adopt these numbers instead of those above determined.

It may not be amiss likewise to add, that the number 12,630 approaches nearer to the mean strength of direct cohesion, as determined from all our vertical experiments, than 13000; which latter was only found to obtain in the pieces or fragments of the battens from which the triangular pieces were cut after the experiments Nos. 3 and 4, Art. 71, had been made upon them; and the reason for selecting these particular pieces for our principal data, has been already explained, (Art. 121.)

We must, however, again revert to a caution before given; viz. that the above numbers are not calculated for furnishing data upon which computations may be generally founded, the specimens from which they have been drawn having been of a very superior quality, selected merely for the sake of the uniformity they were likely to give, in order thence to deduce the theory which might be applicable to other cases.

142. Considering this point now established, I was anxious to prosecute such a series of experiments as might supply the necessary numbers for every useful species of wood; and, as his majesty's dock-yard, Woolwich, furnished an excellent opportunity, I solicited permission of *the Honourable the Principal Officers and Commissioners of his Majesty's Navy*, to allowed to make such a selection from the above-mentioned dock-yard as would best answer my

intended purpose; and having obtained the most liberal and unlimited permission to pursue my inquiries, agreeably to my request, I selected (assisted by the extensive practical knowledge of the qualities of timber of Mr. Hookey, and other gentlemen* in the

* I ought, in particular, in this place, to acknowledge my obligations to T. Browne, Esq. Master-Attendant in the above dock, for his obliging attention, in procuring specimens of the Mar forest fir; for there being none of that wood in his Majesty's yard, this gentleman had three timbers purposely sent for from Scotland; one of which had been felled for three years, another one year, and the third eight months; from which the specimens were cut, as reported in the following table; Nos. 47, 48, and 52, being converted from the first of the trees above mentioned, No. 46 from the sap of the second, and the others from the third tree.

Each of these trees was about 28 inches in diameter at the butt, and contained 50 feet in length of serviceable timber, the grain remarkably clean, free from knots, and full of turpentine: and, from the results in the following table, it appears that the strength exceeds that of any other fir that was submitted to experiment, although the second specimen of Riga was selected from a tree supposed to be of superior quality, on purpose to form the comparison.

The forest of Mar, from which these timbers were brought, the property of the Earl of Fife, is 20 miles in length, and, in some places, 20 miles in breadth; and contains upwards of 60,000 trees of the above description, besides an immense number of less dimensions, fit for building, and various other purposes.

The management of this very extensive forest appears to have been formerly much neglected; but great care is now taken to promote the growth of many thousand young timbers, which will follow in succession those at present fit for the axe. The forest of Mar is now the only remaining one, of any considerable extent, of the ancient forests of Scotland; the woods of it, as will appear from the following table, are of very superior quality; which, together with its immense extent, renders it an object of great national importance.

The *larch*, of which there are four specimens, is also the growth of Scotland, the property of his Grace the Duke of Atholl. This timber is subject to knots; and, when any of these fall near the centre of fracture, in small specimens, such as those in the following

yard,) the specimens, of which the results are stated in the following table, and which may be generally considered as exhibiting the medium strength of the several woods there enumerated.

The pieces were each 8 feet in length and 2 inches square, after planing; but they were rested on props 7 feet apart, excepting only a few second specimens, which were broken at 6 feet.

Explanation of the Table of Data.

143. In the following table, the 1st column contains the number of the experiments; the 2d, the names of the woods, and their dimensions; and the 3d, their respective specific gravities at the time of making the experiments.

The 4th and 5th columns contain—the former the greatest weight, and the latter the corresponding deflection while the elasticity remained perfect. These numbers were found by loading the scale successively with different weights, and observing the deflection for each; and, as soon as it was found that the deflection increased in a greater ratio than the weights, the operation was discontinued, and the previous weight and deflection registered, as in the table: the piece was then turned with that side horizontal that was before vertical, and the deflec-

table, they diminish the strength very considerably; and it is on this account our first two sets turned out so weak; the third and fourth specimens were perfectly free from knots; and the strength of the wood in large scantlings ought to be estimated by the latter results. The last specimen was from a tree felled in April, 1802, and had been lying for six years under a stack of timber, and deprived of the air during the whole of that time, without experiencing any apparent diminution of its strength.

tion again obtained in the same manner: we have thus two results for the elasticity of each piece.

The 6th column contains the weight under which the piece broke; and the 7th, the ultimate deflection at the moment of the fracture.

The 8th column shews the depth of the neutral axis from the upper side of the beam; which was not, in all cases, deduced from the experiment itself; as, in long pieces, the indications of this point are not always sufficiently distinct; but, in these cases, the fragments were again broken at a short length, in order to leave no doubt in this respect.

The 9th column contains the value of U for finding the ultimate deflection from the formula $\frac{l^2}{d\,\Delta} = \mathrm{U}$. (Art. 103-7.)

The 10th column contains the value of the elasticity, as determined from the formula $\mathrm{E} = \frac{l\,\acute{\mathrm{W}}}{a\,d^3\,\delta}$. (Art. 103-1.)

The 11th contains the value of the transverse strength from the formula $\mathrm{S} = \frac{l\,\mathrm{W}}{4\,a\,d^2}$; which is near enough for most practical cases. (Art. 115-4.)

But this value is given more accurately in the 12th column, from the formula $\acute{\mathrm{S}} = \frac{l\,\mathrm{W}\sec^2\Delta}{4\,a\,d^2}$ (Art. 115-2.)

And the 13th and last column shews the absolute value of direct cohesion on a square inch from the formula $\mathrm{C} = \frac{\acute{\mathrm{S}}\,d^2}{(d-\mathrm{D})^2}$.

This latter formula is obtained as follows: the strain being as before found, equal to $\frac{l\,\mathrm{W}\sec^2\Delta}{4}$; and half this being due to tension, and half to com-

pression, we shall have, in the breaking state, $\frac{l W \sec^2 \Delta}{8} = \frac{1}{2} a (d - D)^2 \times C$, D being the depth of the neutral axis; and, consequently, $d - D =$ the depth of tension, and C the strength of direct cohesion on a square inch. (Art. 138.)

But, $\frac{l W \sec^2 \Delta}{4 a d^2} = \acute{S}$; whence $\frac{1}{2} a d^2 \acute{S} = \frac{1}{2} a (d - D)^2 C$:

$$\text{consequently, } C = \frac{\acute{S} d^2}{(d - D)^2}.$$

And, by the reverse of the above, $\acute{S}$ may be found when C and D are given; for we have, then,

$$\acute{S} = \frac{(d - D)^2 C}{d^2}.$$

(*Copy of a Report transmitted to the Hon. the Principal Officers and Commissioners of His Majesty's Navy.*)

TABLE OF DATA,

CONTAINING THE

Results of Experiments on the Elasticity and Strength of various Species of Timber: selected from His Majesty's Dock-yard, Woolwich.

No. of Experiments.	*Names of the Woods, and Dimensions.*	*Specific Gravity.*	*Greatest weight and deflection while the elasticity remained perfect.* *Weight, in lbs.*	*Deflection, in inches.*	*Breaking weight, in lbs.*	*Ultimate deflection, in inches.*	*Depth of neutral axis, in inches.*	*Value of U, from the Formula* $U=\frac{l^2}{d\,\Delta}$.	*Value of E, from the Formula* $E=\frac{l^3\,W}{a\,d^3\,\delta}$.	*Value of S, from the Formula* $S=\frac{l\,W}{4\,a\,d^2}$.	*Value of S′, from the Formula* $S'=\frac{l\,W\sec^2\Delta}{4\,a\,d^2}$.	*Value of direct cohesion on a square inch, from the Formula* $C=\frac{S'\,d^2}{(d-D)^2}$.
1	Teak, 7 ft. by 2 in. sq.	742	300 300	1·065 1·093	1020	4·75	1·2					
2		749	300 300	1·150 1·130	975	4·20	1·2					
3		744	300 300	1·276 1·192	820	4·00	...					
	Mean Results......	745	300	1·151	938	4·32	1·2	818	9657802	2462	2488	15550
4	Poon, 7 ft. by 2 in. sq.	600	150 150	·830 ·780	860	6·00	1·25					
5		570	150 150	·820 ·837	848	5·75	...					
6		568	150 150	·837 ·830	830	6·00	1·20					
	Mean Results	579	150	·822	846	5·92	1·225	596	6759200	2221	2266	14787

TABLE—*Continued.*

No. of Experiments.	Names of the Woods, and Dimensions.	Specific Gravity.	Greatest weight and deflection while the elasticity remained perfect. Weight, in lbs.	Deflection, in inches.	Breaking weight, in lbs.	Ultimate deflection, in inches.	Depth of neutral axis, in inches.	Value of U, from the Formula $U=\frac{l^2}{d\ \Delta}$.	Value of E, from the Formula $E=\frac{l^3\ \acute{W}}{a\ d^3\ \delta}$.	Value of S, from the Formula $S=\frac{l\ W}{4\ a\ d^2}$.	Value of $\acute{S}$, from the Formula $\acute{S}=\frac{l\ W\ sec^2\ \Delta}{4\ a\ d^2}$.	Value of direct cohesion on a square inch, from the Formula $C=\frac{\acute{S}\ d^2}{(d-D)^2}$.
7	English Oak,	988	150	1·420	470	6·00	1·3					
	1st specimen,		150	1·420								
8	7 ft. by 2 in. sq.	998	150	1·700	421	5·90	1·3					
	inferior specimen,		150	1·700								
9		925	150	1·650	460	5·80	1·3					
			150	1·650								
	Mean Results	969	150	1·590	450	5·90	1·3	598	3494730	1181	1205	9336
10	English Oak,	942	200	1·260	640	7·90	1·2					
	Do. 2d specimen,		200	1·280								
11	6 ft. by 2 in. sq.	900	200	1·290	623	8·30	1·2					
	reduced to 7 feet.		200	1·290								
12		960	200	1·275	649	8·10	1·2					
			200	1·285								
	Mean Results	934	200	1·280	637	8·10	1·2	435	5000200	1672	1736	10853
13	Canadian Oak,	865	225	1·150	660	5·70	1·1					
	7 ft. by 2 in. sq.		225	1·150								
14		885	225	1·009	708	6·20	...					
			225	1·011								
15		867	225	1·070	651	6·10	1·15					
			225	1·070								
	Mean Results	872	225	1·080	673	6·00	1·125	588	8595864	1766	1083	11428

TABLE—*Continued.*

No. of Experiments.	Names of the Woods, and Dimensions.	Specific Gravity.	Greatest weight and deflection while the elasticity remained perfect.		Breaking weight, in lbs.	Ultimate deflection, in inches.	Depth of neutral axis, in inches.	Value of U, from the Formula $U=\frac{l^2}{d\,\Delta}$.	Value of E, from the Formula $E=\frac{l^3 W}{a\,d^3\,\delta}$.	Value of S, from the Formula $S=\frac{l\,W}{4\,a\,d^2}$.	Value of S', from the Formula $S'=\frac{l\,W\sec^2\Delta}{4\,a\,d^2}$.	Value of direct cohesion on a square inch, from the Formula $C=\frac{S'\,d^2}{(d-D)^2}$.
			Weight, in lbs.	Deflection, in inches.								
16	Dantzic Oak, 7 ft. by 2 in. sq.	767	200 200	1·710 1·690	520	5·00	1·2					
17		787	200 200	1·260 1·300	580	4·10	1·2					
18		713	200 200	1·855 1·715	580	5·50	...					
	Mean Results ...	756	200	1·590	560	4·86	1·2	724	4765750	1457	1477	7386
19	Adriatic Oak, 7 ft. by 2 in. sq.	941	150 150	1·070 1·070	560	6·00	1·20					
20		948	150 150	1·550 1·450	500	5·50	1·25					
21		1090	150 150	1·720 1·720	520	5·70	1·15					
	Mean Results ...	993	150	1·430	526	5·73	1·2	610	3885700	1383	1409	8808
22	Ash, 7 ft. by 2 in. sq.	760	225 225	1·270 1·250	777	9·00	1·35					
23		758	225 225	1·300 1·270	760	9·10	1·30					
24		762	225 225	1·240 1·270	780	8·66	1·25					
	Mean Results ...	760	225	1·266	772	8·92	1·3	395	6580750	2026	2124	17337

TABLE—*Continued.*

No. of Experiments.	Names of the Woods, and Dimensions.	Specific Gravity.	Greatest weight and deflection while the elasticity remained perfect.		Breaking weight, in lbs.	Ultimate deflection in inches.	Depth of neutral axis, in inches.	Value of U, from the Formula $U=\frac{l^2}{d\,\Delta}$.	Value of E, from the Formula $E=\frac{l^3\,W}{a\,d^3\,\delta}$.	Value of S, from the Formula $S=\frac{l\,W}{4\,a\,d^2}$.	Value of $\acute{S}$, from the Formula $\acute{S}=\frac{l\,W\sec^2\Delta}{4\,a\,d^2}$.	Value of direct cohesion on a square inch, from the Formula $C=\frac{\acute{S}\,d^2}{(d-D)^2}$.
			Weight, in lbs.	Deflection, in inches.								
25	Beech, 7 ft. by 2 in. sq.	712	150	1·075	565	6·00	1.2					
			150	1·025								
26		628	150	1·009	600	5·70	...					
			150	1·024								
27		638	150	1·025	615	5·50	1·2					
			150	1·000								
	Mean Results....	696	159	1·026	593	5·73	1·2	615	5417266	1556	1586	9912
28	Elm, 6 ft. by 2 in. sq. reduced to 7 feet.	583	125	1·620	368	7·00	1·2					
			125	1·610								
29		540	125	1·420	398	6·93	1·1					
			125	1·460								
30		535	125	2·070	394	6·86	...					
			125	1·930								
	Mean Results....	553	125	1·685	386	6·93	1·15	509	2799347	1013	1042	5767
31	Pitch Pine, 7 ft. by 2 in. sq.	712	150	1·133	650	6·25	1·2					
			150	1·166								
32		628	150	1·140	595	5·75	1·2					
			150	1·110								
33		641	150	1·166	620	6·00	...					
			150	1·091								
	Mean Results....	660	150	1·134	622	6·00	1·2	588	4900466	1632	1666	10415

TABLE—*Continued.*

No. of Experiments.	Names of the Woods, and Dimensions.	Specific Gravity.	Greatest weight and deflection while the elasticity remained perfect. Weight, in lbs.	Deflection, in inches.	Breaking weight, in lbs.	Ultimate deflection in inches.	Depth of neutral axis, in inches.	Value of U, from the Formula $U=\frac{l^2}{d\ \Delta}$.	Value of E, from the Formula $E=\frac{l^3\ W}{a\ d^3\ \delta}$.	Value of S, from the Formula $S=\frac{l\ W}{4\cdot a\ d^2}$.	Value of $\acute{S}$, from the Formula $\acute{S}=\frac{l\ W\ sec^2\ \Delta}{4\ a\ d^2}$.	Value of direct cohesion on a square inch, from the Formula $C=\frac{\acute{S}\ d^2}{(d-D)^2}$.
34	Red Pine, 7 ft. by 2 in. sq.	655	150 150	·825 ·825	473	5·70	1·3					
35		667	150 150	·700 ·725	530	5·83	1·25					
36		650	150 150	·725 ·730	530	5·96	1·25					
	Mean Results....	657	150	·755	511	5·83	1·26⅔	605	7359700	1341	1368	10000
37	New England Fir, 7 ft. by 2 in. sq.	560	150 150	·862 ·862	446	4·50	1·36					
38		560	150 150	·970 ·970	403	4·70	1·30					
39		540	150 150	·960 ·960	411	4·78	1·33					
	Mean Results....	553	150	·931	420	4·66	1·33	757	5967400	1102	1116	9947
40	Riga Fir, 1st specimen, 7 ft. by 2 in. sq.	730	125 123	·812 ·837	420	5·80	1·35					
41		765	125 125	·912 ·912	440	6·10	1·33					
42		763	125 125	·837 ·910	406	6·10	...					
	Mean Results....	753	125	·870	422	6·00	1·35	586	5314570	1108	1131	10707

No. of Experiments.	Names of the Woods, and Dimensions.	Specific Gravity.	Greatest weight and deflection while the elasticity remained perfect.		Breaking weight, in lbs.	Ultimate deflection, in inches.	Depth of neutral axis, in inches.	Value of U, from the Formula $U=\frac{l^2}{d\,\Delta}$.	Value of E, from the Formula $E=\frac{l^3\,W}{a\,d^3\,\delta}$.	Value of S, from the Formula $S=\frac{l\,W}{4\,a\,d^2}$.	Value of S', from the Formula $S'=\frac{l\,W\sec^2\Delta}{4\,a\,d^2}$.	Value of direct cohesion on a square inch, from the Formula $C=\frac{S'\,d^2}{(d-D)^2}$.
			Weight, in lbs.	Deflection, in inches.								
43	Riga Fir, 2d specimen, 6 ft. by 2 in. sq.	714	150 150	·794 ·794	567	5·50	......					
44		768	150 150	·907 ·909	367	6·00	......					
45		732	150 150	·950 ·950	467	6·50	......					
	Mean Results....	738	150	·883	467	6·00	......	......	3962800	1051	1081	......
46	Mar Forest Fir, 1st specimen, 7 ft. by 2 in. sq.	715	125 125	1·560 1·500	360	5·50	1·3					
47		616	125 125	1·370 1·250	463	5·50	1·3					
48		684	125 125	1·370 1·370	465	7·00	1·3					
49		769	125 125	1·560 1·560	457	6·00	1·3					
	Mean Results....	696	125	1·442	436	6·00	1·3	588	2581490	1144	1166	9539
50	Mar Forest, 2d specimen, 6 ft. by 2 in. sq.	720	150 150	1·150 1·150	600	7·00	1·3					
51		756	150 150	1·250 1·150	517	6·00	1·3					
52		603	150 150	0·675 0·675	567	6·25	......					
	Mean Results....	693	150	1·006	561	6·42	1·3	403	3478328	1262	1310	10691

TABLE—*Continued.*

No. of Experiments.	Names of the Woods, and Dimensions.	Specific Gravity.	Greatest weight and deflection while the elasticity remained perfect. Weight in lbs.	Deflection in inches.	Breaking weight, in lbs.	Ultimate deflection, in inches.	Depth of neutral axis, in inches.	Value of U, from the Formula $U=\frac{l^2}{d\,\Delta}$.	Value of E, from the Formula $E=\frac{l^3\,W'}{a\,d^3\,\delta}$.	Value of S, from the Formula $S=\frac{l\,W}{4\,a\,d^2}$.	Value of S', from the Formula $S'=\frac{l\,W\sec^2\Delta}{4\,a\,d^2}$.	Value of direct cohesion on a square inch, from the Formula $C=\frac{S'\,d^2}{(d-D)^2}$.
53	Mar Forest, 3d specimen, 6 ft. by 2 in. sq.	700	150 150	1·150 1·150	561	6·5	1·3					
54		710	150 150	1·230 1·170	570	6·5	1·3					
55		698	150 150	0·675 0·675	552	6·25	1·3					
	Mean Results....	703	150	1·006	561	6·42	1·3	403	3478328	1262	1310	10691
56	Larch, 1st specimen, 7 ft. by 2 in. sq.	504	125 125	1·930 1·910	300	8·60	...					
57		576	125 125	1·740 1·760	340	8·60	...					
58		514	125 125	1·970 2·000	336	8·54	...					
	Mean Results....	531	125	1·885	325	8·58	...	411	2465433	853	890	...
59	Larch, 2d specimen, 6 ft. by 2 in. sq.	552	125 125	0·750 0·750	300	6·00	...					
60		480	125 125	0·812 0·812	412	4·50	...					
61		534	125 125	0·875	398	4·50	...					
	Mean Results....	522	125	0·812	370	5·00	...	513	3591130	832	850	...

No. of Experiments.	Names of the Woods, and Dimensions.	Specific Gravity.	Greatest weight and deflection while the elasticity remained perfect.		Breaking weight, in lbs.	Ultimate deflection, in inches.	Depth of neutral axis, in inches.	Value of U, from the Formula $U = \frac{l^2}{d \Delta}$.	Value of E, from the Formula $E = \frac{l^3 W}{a d^3 \delta}$.	Value of S, from the Formula $S = \frac{l W}{4 a d^2}$.	Value of $\acute{S}$, from the Formula $\acute{S} = \frac{l W \sec^2 \Delta}{4 a d^2}$.	Value of direct cohesion on a square inch, from the Formula $C = \frac{\acute{S} d^2}{(d - D)^2}$.
			Weight in lbs.	Deflection in inches.								
62	Larch, 3d specimen, 6 ft. by 2 in. sq.	546	150 150	0·750 0·750	417	4·70	1·25					
63		552	150 150	0·825 0·825	497	4·90	1·25					
64		552	150 150	0·750 0·750	537	5·00	1·20					
65		576	150 150	1·050 ·950	552	5·40	1·20					
	Mean Results....	556	150	0·831	501	5·00	1·225	518	4210830	1127	1149	7655
66	Larch, 4th specimen, 6 ft. by 2 in. sq.	552	150 150	·831 ·831	500	4·8	1·2					
67		581	150 150	·900 ·864	515	5·2	1·2					
68		548	150 150	·762 ·798	515	5·0	1·2					
	Mean Results....	560	150	·831	510	5·0	1·2	518	4210830	1149	1172	7352
69	Norway Spar, 6 ft. by 2 in. sq.	600	200 200	·800 ·800	667	4·0	1·35					
70		600	200 200	·760 ·740	617	4·0	1·25					
71		580	200 200	·840 ·860	680	4·0	1·30					
	Mean Results....	577	200	·800	655	4·0	1·30	648	5832000	1474	1492	12180

To the Hon. the Principal Officers and Commissioners of His Majesty's Navy.

Experiments on the Strength of Bent Timber.

144. In naval architecture it is always necessary to make use of a deal of bent timber; which, as far as can be done, is selected out of natural grown pieces, as nearly as possible of the required form, and is commonly known in the dock-yards by the term *compass timber,* which was formerly contracted for at a higher rate than that of straight growth: but both compass and straight timber is now, I believe, sent in at the same price. The great call for the former, however, during the war, rendered it very scarce, and much time and labour were employed in examining the stacks, in order to select pieces proper for each required purpose; and as the pieces, when they could be obtained, generally exceeded the requisite dimensions, much was necessarily cut away, and a great difference was always found between the *first* and the *converted* contents: the pieces were also, frequently, very much grain cut, which necessarily diminished their strength very considerably.

These inconveniences, and particularly the great difficulty in obtaining compass timber, led Mr. Hookey, at that time master boat-builder in Woolwich dock-yard, but now assistant builder, to extend a method which he had long practised, of bending boat timbers, to the bending of the largest ship timbers; and, having obtained permission to have a machine constructed for the purpose, it was found to answer every possible expectation that

could be formed of it; the largest timbers, viz. pieces 18 inches square, being brought to any required curve in about 15 minutes after being placed upon the machine: a description of which, in its original state, (but it has since received some improvements,) may be seen in vol. XXXII of the *Transactions of the Society of Arts.*

The method of preparing the timber is as follows: —a fine saw cut is made from one end, or both, according to the form into which the timber is to be bent; the length of it being also different, according to the length of the piece and the degree of curvature: but commonly, in a curve, the height of which is about $\frac{1}{6}$th or $\frac{1}{8}$th of the whole length, the saw cut from each end is about $\frac{1}{3}$d of the length. The piece is then boiled for some hours, depending upon its lateral dimensions, and placed upon the machine, when the screws, &c., being applied, the required curvature is obtained, as above stated, in about 12 or 15 minutes; after which it is screw bolted, and is then ready for use. The reader, by referring to *figs.* 6 and 7, *plate* III., will readily understand the above description; these figs. representing the fragments of two pieces bent for the following experiments. It is only necessary to observe, that the keys *k*, *k*, and K, are no part of the original plan, but were suggested during our experiments.

The advantages attending this method of bending timber for the purposes of ship building, are, 1st, That it dispenses with the use of compass timber, which is become very scarce; and, therefore, no impediment would arise to the service, if the necessary quantity of timber of this kind could not be in any way procured. 2dly, It saves a deal of the time

and labour necessary for unstacking and restacking piles of timber, to procure pieces of requisite compass; any piece of the proper length and squarage being at once available with the application of the machine. 3dly, It saves a great quantity of timber, which is necessarily cut to waste, in bringing compass timber to its required dimensions; the conversion, in some cases, taking away a considerable part of the original contents; while, in bending timber, the original and converted contents are nearly the same. But, notwithstanding these recommendations in its favour, there appears to be a prejudice, well or ill founded, against the adoption of it, and some objections have been offered to the practice: the first of which is, that boiling the timber, and the strain impressed upon it, have a tendency to weaken the pieces, and, consequently, the ship into which such timbers are introduced: and, secondly, that the bolts are not sufficient to keep the two parts in a proper degree of contact, so as to prevent the introduction of damp and moisture. The latter point must be left to the decision of the practical builder; but with regard to the strength, this may be otherwise determined, and I therefore solicited permission of the Navy Board to be allowed to make experiments on bent pieces of natural growth, grain cut, and others, bent on the principle of Mr. Hookey, and the results of these experiments will be seen in the following table: from which it will appear, that, taking the medium between the natural grown pieces and those which are partly so and partly grain cut, no defect in point of strength will be found on the side of those bent upon the above plan. I also wished to try what effect boiling

and steaming timber had upon the ultimate strength without bending; the account of which is given in my third report. From which it appears, that, although there is an obvious falling off in the strength of those pieces boiled for a long time, the defect is very small while the boiling or steaming is not continued beyond the proportion of an hour to an inch in thickness, which is the usual practice in the dock-yard.

(Copy of a Report transmitted to the Honourable the Principal Officers and Commissioners of His Majesty's Navy, containing)

145. ***Experiments on the Strength of Bent Oak Scantlings: 1st, Of Natural Growth; 2dly, Grain cut; and, 3dly, On those bent according to the Plan of Mr. Hookey. The latter with a Saw cut, and without it. Also, the former of these with and without Keys.***

Note.—The pieces were each 6 feet long and 2 inches square, but they were broken on props 5 feet apart.

No. of Experiments.	*Nature of the Pieces.*	*Arch up or down.*	*First Curve, in inches.*	*Specific Gravity.*	*Breaking Weight.*	*Last Deflection below the Props.*	*Strength computed from the Formula* $S=\frac{l\,W\,\sec^2\Delta}{a\,d^2}$.
1	Natural Growth	up	6	804	680	— 2	5250
2	Natural Growth	up	8	820	764	— 0	6016
3	Do.	down	6	822	768	10	6400
4	Do.	down	8	874	762	13	6630
5	Grain cut	up	7½	960	585	— 3	4643
6	Do.	up	8½	830	568	— 2	4489
7	Do.	down	7½	938	546	10	4549
8	Do.	down	8½	840	550	10	4583
9	Bent whole	up	7½	798	667	— 1	5257
10	Do.	down	7½	810	617	13	5413
11	Saw kerf, but no keys	up	8½	886	517	+ 2	——
12	Saw kerf, but no keys	down	8½	856	517	15	——
13	Saw kerf, with square keys	up	8½	754	712	+ 2	5628
14	Saw kerf, with square keys	down	8½	732	662	14	5880
15	Saw kerf, with cylindrical keys	up	6	873	717	+ 5	5789
16	Saw kerf, with cylindrical keys	down	6	873	762	12¾	6629

Note 1.—The last deflection, having the sign *plus* + prefixed, indicates that the pieces arched so many inches the contrary way before breaking: and those marked *minus* —, wanted the number of inches following, of coming down to the level of the props.

Note 2.—The pieces laid with the arch up were necessarily supported by the outside of the props; these, therefore, must be considered as being broke at 5 feet 3 inches, which was the distance from the outside of one prop to that of the other; and this is the case even where the pieces bent the contrary way; for, notwithstanding the middle of the piece came below the props, the half-lengths were still sufficiently curved to throw the principal bearing on the outside.

In each of the *figs.* 6 and 7, *plate* III., A B C D represents a fragment of the scantlings; *a a*, *b b*, *c c*, the screw bolts, and *m n* the saw cut; which latter is two feet, or one-third the length of the piece. In *fig.* 7, K represents the form of the key, which was of oak, an inch long and $\frac{1}{2}$ an inch deep, let in $\frac{1}{4}$ of an inch into each part; and, in *fig.* 6, *k* and *k* are copper bolts, of $\frac{1}{2}$-inch diameter; which, therefore, also laid $\frac{1}{4}$ of an inch into each part; and in both figures the keys passed through the whole thickness of the scantling.

The idea of this mode of keying was suggested in our first experiments on pieces of this description; viz. Nos. 11 and 12, in which it was found that the screw bolts were not sufficient to prevent the part above and below *m n* from sliding upon one another. This defect may not have place when pieces of this kind are introduced into a ship, in consequence of the number of tree-nails with which the futtocks are pierced, which have necessarily a tendency to prevent that slipping of the parts noticed above. But, even in this case, I am convinced that considerable stiffness would be gained by keying the pieces after the manner of *fig.* 6, where it may be observed, that hard wood, as sound oak or lignum vitæ, would answer equally as well as copper bolts; and farther, that as the neutral axis in any section of fracture is generally at about $\frac{3}{8}$ths of the depth, there would *be no loss of strength* in the piece, provided the key did not exceed $\frac{1}{4}$th of the whole depth.

N.B. Mean strength { Nos. 1, 2, 3, 4, of natural growth, 743 }
{ Nos. 13, 14, 15, 16, bent and keyed, 713 }

Additional Experiments.

146. In order to form a comparison between the strength of a piece of timber bent upon Mr. Hookey's principle, and a straight piece in its natural state, two pieces were formed from the same scantling, having been only parted by the saw; the bent piece was brought to a curve of 9½ inches, and keyed, as in *fig.* 6, *plate* III.; the two pieces were then broken at the same distance, viz. 5 feet; their other dimensions being also the same as those above. The results of these experiments are as follow:

Straight piece, not boiled,	deflected 5½ inches; broke with 667 lbs.
Bent to a curve of 9½ in., arch down,	deflected to 14½ inches; broke with 727 lbs.

By a comparison with all the above results, we obtain the following proportional breaking weights; viz.

	lbs.
Natural growth	743
Bent on Hookey's principle, and keyed......	730
Bent, without a saw cut	632
Grain cut..................................	562
Bent on Hookey's plan, without keys	517
Straight, and in natural state, deduced from the results of the 2d specimen of the first report	764

Note.—In comparing the first two of the above numbers with the last, it should be remembered, that although the former were broken with less weight, it does not indicate a less degree of strength; the same weight producing a greater strain upon a bent than upon a straight piece, proportional to the secant squared of the angle of deflection.

To the Hon. the Principal Officers and Commissioners of His Majesty's Navy.

147. Copy of a Report transmitted to the Honourable the Principal Officers and Commissioners of His Majesty's Navy; containing

Experiments on the Strength of Oak Timber, in its natural State, compared with similar pieces boiled and steamed for different Periods.

Note.—The following pieces of oak were all cut from the same log, the mean specific gravity of which was 822.

No. of Experiments.	*Boiled or Steamed.*	*No. of Hours.*	*Length, in feet.*	*Square, in inches.*	*Deflection with 100 lbs.*	*Ultimate Deflection.*	*Breaking Weight, lbs.*	*Mean Breaking Weight, lbs.*
1	Natural state	0	6	2	·425	6·0	722	669
2		0	6	2	·500	6·5	617	
3	Steamed	5	6	2	·450	6·0	617	669
4	Steamed	5	6	2	·425	7·0	722	
5	Steamed	10	6	2	·430	6·0	662	614
6	Steamed	10	6	2	·475	5·0	567	
7	Boiled	2	6	2	·500	5·0	567	614
8	Boiled	2	6	2	·425	6·5	662	
9	Boiled	4	6	2	·462	7·5	662	614
10	Boiled	4	6	2	·525	4·0	567	
11	Boiled	6	6	2	·550	6·0	597	589
12	Boiled	6	6	2	·425	5·5	582	
13	Boiled	8	6	2	·475	5·5	647	639
14	Boiled	8	6	2	·500	7·0	632	
15	Boiled	10	6	2	·550	5·5	567	607
16	Boiled	10	6	2	·500	6·0	647	

Nos. 17 and 18, bent and keyed on Hookey's plan, part of the same log, and broke at the same length, viz. 6 feet; and the same squarage, viz. 2 inches.

17	Boiled	3	1st curve 10 in.	Arch up	Breaking wt. 632
18	Boiled	3	1st curve 10 in.	Do. down	Breaking wt. 636

To the Hon. the Principal Officers and Commissioners of His Majesty's Navy.

There is not in the above experiments that degree of uniformity that we might have expected, considering the pieces were all cut from the same log. It should be observed, however, that the two experiments, 11 and 12, ought not to be considered as equally conclusive with the others, as they each broke at a knot about 6 inches from the centre of the beam.

Rejecting these, therefore, there appears, generally, to be a slight loss in strength from boiling and steaming; but it is not very perceptible while that process is not continued beyond the time usually allowed in the dock-yards.

In several experiments which I made on pieces boiled only for 2 or 3 hours, there was no apparent defect in strength; some of them even exceeding, and others falling a little short of similar unboiled pieces: but as they were not all from the same timber, they would not, probably, be thought conclusive if they were detailed: on which account they are omitted.

On Trussed Girders.

148. We shall now conclude this course of experiment with the four following, on girders, trussed and plain; the two former, viz. No. 1 and No. 3, were very accurately made, and constructed on a scale of 2 inches to the foot, from the drawing given by Nicholson (*plate* XXXIX., "*Carpenter's New Guide*"); the former being supposed to denote a 34-feet, and the other a 25-feet girder.

EXPERIMENTS

On the Deflection and Strength of Girders, trussed and plain.

No. of Experiments.	*Distance between the Props.*	*Depth of the Girder.*	*Breadth of the Girder.*	*Weight in lbs.*	*Deflection in inches.*	*REMARKS.*
1	ft. in. 5 8	ft. in. 0 2	ft. in. 0 1⅞	100 200 300 400 450 500	·35 ·67 1·05 1·47 1·75 2·25	Truss in 3 pieces; length of centre piece 1 ft. 6 in. Distance of extreme butments, 4 ft. 10 in. 2 king bolts, 2 plate bolts, and 5 screw bolts.
2	ft. in. 5 8	ft. in. 0 2	ft. in. 0 1⅞	100 200 300 400 450 500	·30 ·60 ·90 1·20 1·35 1·55	Without a truss.
3	ft. in. 4 2	ft. in. 0 2	ft. in. 0 1⅞	100 200 400 600 700 743 803 903 953	·15 ·30 ·57 ·87 1·20 1·30 1·45 1·50 broke	Truss in 2 pieces; distance of extreme butments, 3 ft. 4 in. 1 king bolt, 2 plate bolts, and 4 screw bolts.
4	ft. in. 4 2	ft. in. 0 2	ft. in. 0 1⅞	100 200 300 400 500 600 717	·15 ·27 ·41 ·57 ·77 1·00 broke	Without a truss.

Nos. 1 and 2 were not broken in the experiment; but it appears that the trussed beam was the weakest; or, at least, it gave the greatest deflections. The wood of No. 1 was certainly inferior to the untrussed beam, but still it was to have been expected that the trusses would have been more than equivalent to the difference in the former respect; but as it was not, the experiment seems to indicate that there is but little efficacy in a truss of that description.

The trusses of No. 3 came fairly into action with each other, and certainly added very considerably to the resistance of the girder.

AN ESSAY

ON THE

STRENGTH AND STRESS OF TIMBER.

PART IV.

APPLICATION OF THE PRECEDING INVESTIGATIONS AND DEDUCTIONS TO PRACTICAL PURPOSES.* BY THOMAS TREDGOLD, CIVIL ENGINEER.

Of the Forms proper to be given to Beams, that they may be of equal Resistance throughout; or be equally liable to break in any part of their Length.

149. It is very important in practice, to know in what proportion the parts of a beam may be reduced without injury to its strength; and in more expensive materials, such as cast and wrought iron, the forms of equal strength are always employed to lessen the expense and weight of structures. Beams of wood are seldom, if ever reduced to the shape of the form of equal strength; but every one must be sensible of the advantage of knowing those points where there is an excess of material, in order

* The Author begs to acknowledge his obligations to Mr. Tredgold, for the following highly useful practical problems.

that the soundest parts may be disposed, as far as can be done in the conversion of the rough material, at the places where the strain is greatest; and that mortises, notches, and the like, may be in places where the strength is in excess.

In treating of the mechanism of the transverse strain, p. 147, it has been shewn that the strain excited at any point of a beam is as the force and the distance of the point of its application from the point strained.

It has also been proved, that the resistance to fracture, is, in similar beams, as the breadth and square of the depth.

In order, therefore, that the ratio between the strain excited, which varies as the force, and the distance of the point of application of that force, or as $w\,x$, and the resistance which varies as $a\,d^2$, may be constant, we must have,

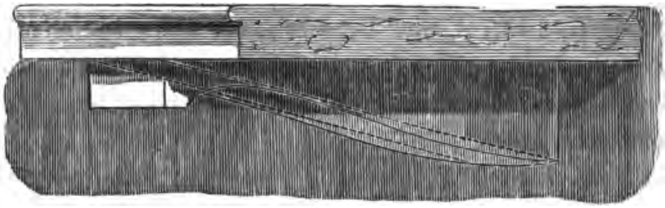
$$a\,d^2 : w\,x :: \mathrm{A} : \mathrm{B};$$

where A is to B as the resistance is to the strain, and must be a constant and determined ratio for every part of the beam.

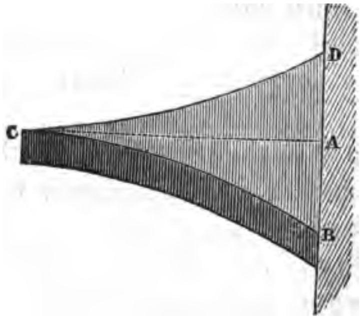
This equality of ratios may be made to obtain in a variety of ways; for example:—

1. We may suppose a the breadth, constant; and the force w to vary, as x; then the d^2 must vary as x^2; or d vary as x. The form of the beam must therefore be that of a wedge; and it applies to the case when a uniformly distributed load is to be supported by a beam fixed at one end. The dotted lines of the annexed figure of a cantilever, exhibits

the form; and whatever form may be considered necessary for ornament, it should not be cut within the lines representing the beam of equal resistance. In this case, half the beam may be cut away, and yet leave it of equal strength in every part.

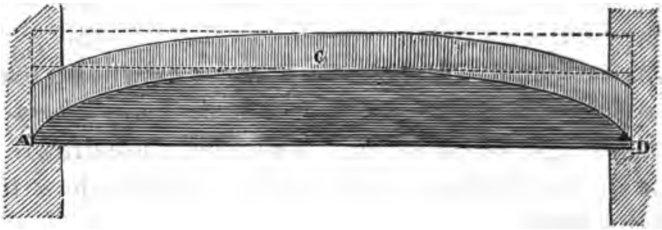


2. If we suppose the depth to be constant, then a must vary as x^2; and the horizontal section should be bounded by a parabola C D, having its vertex at C, and a straight line A C, a tangent to the vertex of the curve; or another parabola, C B.



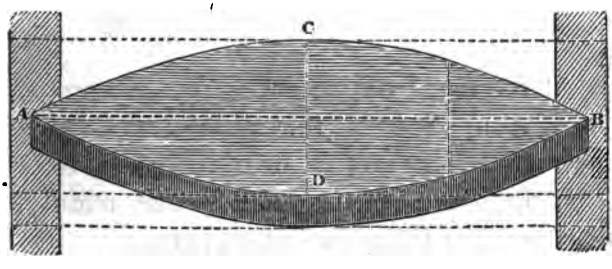
3. When a beam is supported at both ends, and sustains a load uniformly distributed over its length, then the strain at any point, C in its length, is as the rectangle I C × I′ C, Art. 110 (see *fig.* 2, *plate* II.) and w is constant; hence, $a\,d^2$ must be proportional to I C × I C; and making the breadth a, constant, we

have d^2 as : $IC \times I'C$, a property of the ellipse. Consequently, if a beam be every where of the same breadth, the outline of its vertical section must be an ellipsis, as shewn by the figure



4. It has been shewn in Art. 108, that the stress from a weight placed on any point of a beam, is as $IC \times I'C$; hence, the form of equal strength for a load rolling along the length of a beam, is also an ellipsis, A C D; but in this case the figure must be placed with its straight side upwards.

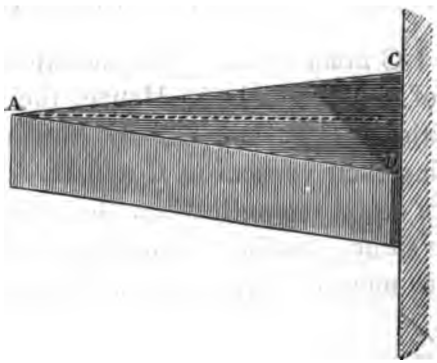
5. If we suppose the depth to be constant, then $a : IC \times I'C$ must be the constant ratio, which is a property of the parabola.* Hence, the outlines of the breadths A C B, and A D B, should be parabolas having their vertices at C and D. And this is the form of equal strength when the depth is every where the same, whether the load be uniformly distributed, or moveable from one part to another.



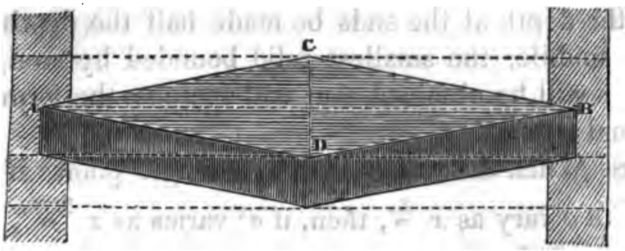
* Hutton's Course, Vol. II. Parabola, Theorem ii. cor.

The cases 3, 4, and 5, shew the forms of equal strength for girders, joists, bressummers, lintels, and other beams, where the chief load is uniform, but which are also subject to a variable load. If the depth be uniform as in case 5, one-third of the beam may be cut away and yet the strength of the parts will be equal. It shews where knots will be least hurtful, and where mortises will do least injury to the strength of the beam. The figures are drawn to the proper curves; and the dotted lines shew the entire beam.

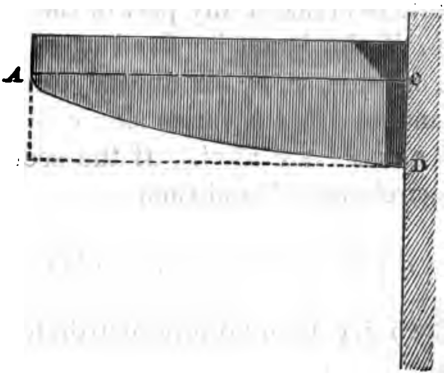
6. If the load be collected at one point, w must be constant; and if we make the depth d constant also, then a must vary as x; that is, the beam must be of the same depth throughout, and its horizontal section will be a triangle, having its base at the wall, if the beam be supported at one end.



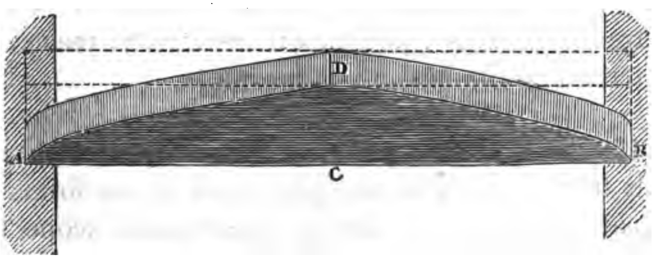
Or, if the beam be supported by two props, and the weight applied in the middle, the horizontal section will consist of two triangles, A D C and B C D, applied base to base in the middle, the vertices being at the points of support.



7. Or the breadth being considered constant, d^2 must vary as x; and in this, the vertical section of the beam must be a parabola, A C D, having its vertex A, at the point where the stress is applied when the beam is fixed at one end.



And when the beam is supported at both ends, the figure must be doubled; the vertices A and B being the points of support.



If the depth at the ends be made half the depth in the middle, the smallest solid bounded by straight lines will be obtained that will include the form of equal strength.

8. When the weight acts at a single point, if we make a vary as $x^{\frac{n}{m}}$, then, if d^2 varies as $x^{\frac{m-n}{m}}$, or d as $x^{\frac{m-n}{2m}}$, we shall always have $a\,d^2$ proportional to x; which is the only condition required to be fulfilled in order to obtain a solid of equal resistance. By giving different values to the exponents m and n, an immense variety of forms may be found that will offer an equal resistance to fracture at any part, or be equally liable to break at any part of their length.

9. Again, if the beam be fixed at one end, and the weight or force, w, be distributed over the length in such a manner that w varies as $c\,x + e\,x^m$, then, $a\,d^2$ varies as $c\,x^2 + e\,x^{m+1}$. If the section of the beam be a circle $a = d$, and then

$$d \text{ varies as } (c\,x^2 + e\,x^{m+1})^{\frac{1}{3}}.$$

When the force is uniformly distributed over the length, then $m = 1$ and d is as $x^{\frac{2}{3}}$ a property of the semi-cubical parabola.

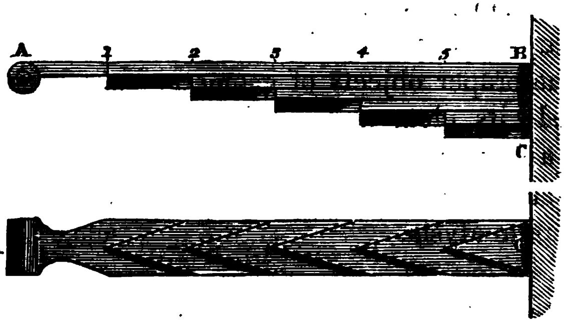
When $m = 2$, then d is as $(c\,x^2 + e\,x^3)^{\frac{1}{3}}$, and the force will be composed of an uniform one, acting with one which increases as the distance from the extremity. This equation will afford the means of determining the best figure for masts.

150. Besides the practical advantages which result from a study of the properties of the forms of equal resistance, it affords considerable scope for

interesting problems, either for geometrical or algebraical analysis. It has been treated geometrically by Galileo, Viviani, and Guido Grandi, (see Opere di Galileo, Tom. III., 1718;) and algebraically by Girard, (Traité Analytique, Sect. II.). Girard has very justly remarked, that the general geometrical solutions of Grandi are very elegant; in fact, they are very fine specimens of the application of geometry, which seem to remind us that it has more power than it is commonly supposed to have; and that in the general cultivation of algebraical reasoning the proper objects of geometry have been overlooked: its clear and tangible evidence, united to such a degree of generalization as is obtained in the problems of Grandi, puts it nearly on a level with the modern analysis.

151. The springs for carriages, and for other purposes, which are formed of thin plates, ought to be of such a form as to render each part of equal strength; for such a form also admits of the greatest degree of flexure with the least injury to the materials. Let the spring A B C, be composed of any number of parallel plates of equal thickness; divide A B into the same number of equal parts, which will give the respective lengths of the springs, as shewn in the figure. For the strength at any point is as the number of thicknesses, and the stress is as the leverage: at the point 1, the leverage is 1, and the thickness 1; at the point 2, the leverage is 2, and the thickness 2; and so on of the rest. The ends of the plates should be of such a form as to bend into a circle; this will be the case when they are triangular and of the same depth at all points;

the curve assumed by the spring, will then be a circle when it is loaded; and the flexure and strength will be the greatest possible. The lower portion of the figure shews the under side of such a spring. If a spring formed of plates of equal thickness, does not bend into a circle, the strain must be irregular upon it; and, consequently, it must be liable to break at certain points.



152. The general condition required to be fulfilled in the formation of springs, is, that all the plates shall bend into curves exactly parallel to the curvature of the external plate: to find the curve, the material must be supposed of equal texture and quality, and it must be the object of the workman to render it as nearly so as possible.

As we have given that form which renders a spring most effective, it will not be necessary to pursue other cases. It will be sufficiently clear that there must be very few cases in practice where the equable flexure, shewn to be the property of a good spring, is procured; and it is from want of knowing distinctly in what the goodness of a spring consists.

When a spring is to be formed of a single plate or bar, the best form is the figure of equal resistance for a constant breadth, or for a breadth increasing, in some proportion not liable to practical objection, toward the point where the straining force is applied. The reason for this construction is, to obtain as much flexure as possible without diminishing the strength.

It has been remarked, that wooden springs are greatly superior to metal ones, in cases where the springs must be allowed to vibrate after performing the requisite action; and the best wood for such purposes is clean straight-grained yellow deal, perfectly free from knots. It is also said, that a metal spring, unless it has something to stop against, to prevent its vibration after its action has ceased, will frequently break in a short space of time where the action is frequently repeated.* This defect of metal springs is very probably owing to their being made too slight; the strain being greater than the elastic force of the metal.

153. SOLUTION OF PRACTICAL PROBLEMS ACCORDING TO THE TABLE OF DATA. PAGE 196.

PROBLEM I.

To determine the Strength of Direct Cohesion of a piece of Timber of any given Dimensions.

Rule.—Multiply the area of the transverse section, in inches by the value of C, in the Table of

* Philos. Mag. II. 67.

Data, (p. 196,) and the product is the strength required.

In practice, the weight the timber has to support should not exceed one-fourth of the strength as calculated by the rule.

Note 1.—If the specific gravity be not the same as the mean tabular specific gravity; say, as the latter is to the former, so is the product by the rule to the correct result. (See Art. 63.)

Note 2.—The line of tension must be directly in the centre of the transverse section, otherwise the timber will not bear so much; and the strength should be calculated by the next problem. (Prob. II.)

Example I.—What weight will be required to tear asunder a piece of teak, 3 inches square, the specific gravity being ·745?

In this case the tabular value of C is....	15550
The area of the section $3 \times 3 =$	9
The weight required	139950 lbs.

Example II.—The diameter of a rod of ash being 2 inches, and its specific gravity ·700, what weight will be required to tear it asunder?

The tabular value of C is............	17337
The area of the section $2 \times 2 \times {\cdot}7854 =$	3·1416
The product..........	54465·9192 lbs. or
54466 lbs nearly.	

Again; as 760 : 700 : : 54466 : 50166 lbs., the answer.

Example III.—What sized square spear of larch should be employed to sustain a load of 6 tons,

where the strain will be in the centre of the transverse section, using the numbers of the third specimen in the Table of Data.

First, 6 tons are equal to............................	13440 lbs.
It must be capable of sustaining 4 times that weight...	4
	53760 lbs.

By reversing the rule, we have 53760 divided by the tabular value of C, or $\frac{53760}{7655} = 7$ inches, the area of the section.

And taking the square root of 7, we have 2·65 inches for the side of the square.

PROBLEM II.

To determine the Strength of a Rectangular Beam, to resist a force drawing it in the direction of the length, when the line of strain does not coincide with the centre of the Transverse Section.

Rule.—Multiply the tabular value of C, by the breadth and square of the thickness of the given beam, both in inches.

Divide this product by the thickness added to 6 times the distance of the line of direction from the centre of the section, in inches; and the quotient will be the strength required.

Note.—The beam may be loaded in practice with one-fourth of the force found by the rule.

Example I.—If two points in a system of framing be connected by a curved piece of timber, of which

the thickness is 5 inches, the breadth 7 inches, and the curvature 4 inches, required the greatest strain it would bear if of English oak similar to our 2d specimen, (p. 197.)

The tabular value of C = ...	10853
The breadth and square of the thickness $7 \times 5^2 =$	175
The thickness added to 6 times the curvature $5 + 6 \times 4$	=29) 1899275
	65492 lbs. answer.

The strain in practice should be $\frac{65492}{4} = 16373$ lbs.

Note 1.—The same rule will apply to find the resistance to compression in the direction of the length under similar circumstances. By this rule the resistance of different parts of ships may be calculated.

Note 2.—In actual construction it is nearly impossible to fit connexions and abutments so accurately that the centre of tension or compression will be in the centre of the piece, even when it is straight, and of equal texture in every part of the section. Hence, for straight pieces, an allowance should be made for the probable deviation. This allowance should not be less than one-third of the thickness, and then, the rule will be—Multiply the transverse section, in inches, by the tabular value of C, and one-third of the product is the strength; and one-twelfth of the product is the strain it may be exposed to in practice.*

* The rule is $\frac{C\,a\,d^2}{d + 6\,\delta} = W$, and when $\delta = \frac{1}{3}d$, it becomes $\frac{C\,a\,d}{3} = W$.

Example II.—To determine the size that should be given to a king-post of Riga fir to support a stress of 16 tons.

First, 16 tons = 35840 lbs., which, multiplied by 12, and divided by the tabular value of C for Riga fir, is $\frac{12 \times 35840}{10707} = 40 \cdot 17$ inches for the area of the section, or very nearly 6·34 inches square.

Example III.—To find the stress that may be put on a short oak strut, the tabular value of C being 10853; and the strut straight, and 6 inches square.

Here, $\frac{6 \times 6 \times 10853}{12} = 32{,}559$ lbs. for the stress required.

PROBLEM III.

To find the Strength of a Rectangular Beam of Timber, fixed at one end, and loaded at the other.

Rule.—Multiply the value of S in the Table of Data, by the area, and the depth of the section in inches, and divide that product by the leverage in inches, and the quotient will be the weight required in lbs.

Note 1.—In case the beam is inclined, the leverage is the distance I L, or $F' L'$, *fig.* 6, *plate* II. When the beam is horizontal, the leverage is usually called the length.

Note 2.—In practice, the load ought not to be greater than one-fourth of the weight found by the rule; for permanent stretching or displacement of the fibres begins to take place as soon as the load

exceeds about one-fourth of the breaking weight. This will be perceived by comparing the weights which the specimens bore without loss of elasticity, with the weights that broke them, in the Table of Data.

Note 3.—If the load be distributed in any manner over the length of the beam, the horizontal distance between the point of support and a vertical line drawn through the centre of gravity of the load, must be taken for the leverage.

Example I.—A beam projecting 5 feet over the point of support, is 6 inches deep and 4 inches in breadth of Riga fir, and is intended to support a load at its extremity; it is required to determine the greatest load it would bear, and the load it may be exposed to without injury.

For Riga fir, $S = 1108$, and the area being $6 \times 4 = 24$, the depth 6 inches, the leverage 5 feet = 60 inches, we have $\frac{1108 \times 24 \times 6}{60} = 2659{\cdot}2$ lbs., the greatest or breaking load; and $\frac{2659{\cdot}2}{4} = 664{\cdot}8$ lbs. for the load it would bear without injury.

Example II.—A cistern to contain 36 cubic feet, or one ton of water, is to be supported by two cantilevers: the projection of the cistern from the face of the wall being 4 feet, it is required to determine the size for the cantilevers.

Let the cantilevers be of larch, such as the 3rd specimen, then we find by the Table of Data $S = 1127$, and the depth 5 inches. The load on them will be 1 ton $= 2240$ lbs., and the weight will be uniformly distributed over the length; therefore, the distances of the centre of gravity from the wall

will be half the length, or 2 feet = 24 inches, which is the leverage. This is the reverse of the preceding operation, on account of the weight being given.

$\frac{2240 \times 4 \times 24}{1127 \times 5} = 38{\cdot}2$ inches nearly, for the area of both cantilevers, or $\frac{38{\cdot}2}{2} = 19{\cdot}1$ inches for the area of one of them; and if the section be rectangular, the depth being 5 inches, the breadth will be 3·82 inches for each cantilever.

PROBLEM IV.

To determine the Strength of a Rectangular Beam of Timber when it is supported at the ends, and is loaded in the middle of its length.

Rule.—Multiply the value of S, in the Table of Data, by 4 times the depth in inches, and by the area of the section in inches, and divide the product by the distance between the supports, in inches, and the quotient will be the greatest weight the beam will bear in lbs.

Note 1.—If the beam be not horizontal, the distance between the supports must be the horizontal distance.

Note 2.—One-fourth of the weight found by the rule should be the greatest weight upon a beam in practice.

Note 3.—If the load be applied at any other point than the middle, it will be as the rectangle of the segments, into which the point divides the distance between the supports, is to the square of half

that distance; so is the weight found by the rule, to the weight the beam will sustain at the given point.

Note 4.—If the load be distributed in any manner whatever over the beam, the centre of gravity of the load must be considered its place, and its stress equal to the whole weight; unless part of such weight be sustained by the supporting points independently of the resistance of the beam.

Example I.—Required the weight a beam of Riga fir, one foot square, would sustain in the middle, its length being 20 feet?

In this case the tabular value of S is 1108, and the depth 12 inches, and the area 144 inches, the length 240 inches; consequently,

$$\frac{1108 \times 4 \times 12 \times 144}{240} = 32010 \text{ lbs.}$$

And the beam may be loaded in practice with $\frac{32010}{4} = 8002\frac{1}{2}$ lbs. without injury to its texture.

If the load were applied at 8 feet distance from the end, instead of being applied in the middle, then it would be 12 feet from the other end; and by note 3, it is $8 \times 12 : 10 \times 10 :: 8002\frac{1}{2} : 8336$ lbs. nearly, for the weight the beam 12 inches square would support at 8 feet from the end; shewing the advantage of applying the load as far from the middle as possible.

Example II.—To determine the size of a girder of Riga fir for a warehouse, where the distance between the points of support is 18 feet, = 216 inches, and the greatest probable stress at the middle, including the weight of the floor itself, 20 tons.

The tabular number is S = 1108, and 20 tons

= 44800 lbs. Let us further suppose that the greatest depth of the timber intended for the purpose is 20 inches. By reversing the rule, we have $\frac{4 \times 44800 \times 216}{1108 \times 4 \times 20 \times 20} = 21 \cdot 83$ inches for the breadth of the girder, which would be obtained by bolting together two pieces, each 20 inches by 11 inches; or much better by putting the two pieces at the most convenient distance apart, that would admit of both resting on the sustaining piece.

If there be only 20 tons distributed uniformly over the surface of the floor, then a girder of 20 inches by 11 inches would be sufficient.

PROBLEM V.

To determine the Dimensions of a Beam, capable of supporting a given Weight with a given degree of Deflection, when supported at one end.

Rule.—Multiply the weight in lbs. by 32; divide the product by the tabular value of E,* multiplied by the breadth and deflection, both in inches; then the cube root of the quotient multiplied by the length in feet, will be the depth required in inches.†

* The value of E in these rules is the tabular value divided by 1728, which renders it unnecessary to reduce the length in feet into inches.

For English Oak, E = 3360
For Riga Fir, E = 3075.

† This rule is applicable to the imperfect fixing which obtains in practice; but the more perfect the mode of fixing is, the nearer the deflection will be to half that determined by the rule, and ordinary cases will usually be a mean between these results.

Example I.—A beam of Riga fir is intended to bear a load of 665 lbs. at its extremity, its length being 5 feet, its breadth 4 inches, and the deflection not to exceed $\frac{1}{4}$ of an inch.

In this case the tabular value of E is 3075; hence, $\frac{665 \times 32}{3075 \times 4 \times \frac{1}{4}} = 6{\cdot}92$; the cube root of which is $1{\cdot}906$; hence, $5 \times 1{\cdot}906 = 9{\cdot}53$ inches, the depth required.

By reference to Example I. of Prob. III. it will be found that a beam of 6 inches depth would be sufficient to bear the load; but when, from the nature of the construction, only a limited degree of flexure can be allowed, this mode of calculation becomes necessary.

Note 1.—When the weight is uniformly distributed over the length of the beam, the deflection will be only $\frac{3}{8}$ths of the deflection from the same weight applied at the extremity; and in the rule the number 12 should be used as a multiplier instead of 32.

Note 2.—If the beam be a cylinder, the deflection will be 1·7 times the deflection of a square beam, other circumstances being the same.

Note 3.—In the above examples the reduction of results to the differences depending on the specific gravity is not shewn, neither is it applicable in practice; but for theoretical comparison it is important, and may always be performed by stating, as the specific gravity of the tabular specimen is to the load supported in any example, so is the actual specific gravity of the specimen to the load it would support under similar circumstances.

PROBLEM VI.

To find the Dimensions of a Beam, capable of supporting a given Weight with a given degree of Deflection, when supported at both ends.

Rule.—Multiply the weight to be supported in lbs. by the cube of the length in feet. Divide this product by the tabular value of E, (see p. 231, note,) multiplied into the given deflection in inches, and the quotient is the breadth multiplied by the cube of the depth in inches.

Note 1.—If the beam be intended to be square, then the breadth is equal to the depth, and the fourth root of the quotient is the depth required.

Note 2.—If the beam be a cylinder, multiply the quotient by 1·7, and then the fourth root will be the diameter of the cylinder.

Note 3.—When the load producing the depression is greater than one-fourth of the greatest stress the beam would bear, it is too great to be trusted in construction; but in timber this limit is seldom exceeded on account of its flexibility.

Note 4.—If the load be uniformly distributed over the length, the deflection will be $\frac{5}{8}$ths of the deflection from the same load collected in the middle. And in the rule, employ $\frac{5}{8}$ths of the weight of the load instead of the whole load.

Example I.—The length of the fir shaft of a water-wheel being 20 feet, and the stress upon it 7 tons, it is required to determine its diameter so that its deflection may not exceed ·2 of an inch.

The tabular value of E = 3075, and 7 tons =

15680 lbs.; hence, (by the rule and note 2) $\frac{1{\cdot}7 \times 15680 \times 20^3}{3075 \times {\cdot}2} = 346730$ nearly. The fourth root of this sum is 24·3 inches, the diameter required.

Shafts which are to be cut for inserting arms, &c. will require to be larger, in a degree equivalent to the quantity destroyed by cutting.

The flexure of shafts ought not to exceed $\frac{1}{100}$ of an inch for each foot in length, this being esteemed the limit; and it will be always desirable to make shafts as short as possible, to avoid bending.

Example II.—The greatest variable load on a floor being 120 lbs. per superficial foot, it is required to determine the depth of a square girder to support it, the area of the floor sustained by the girder being 160 feet, the length of the girder 20 feet, and the deflection not to exceed half an inch.

The value of E for Riga fir is 3075, and the weight is $120 \times 160 = 19200$ lbs. uniformly distributed; hence, (by the rule and note 4) we have $\frac{\frac{5}{8} \times 19200 \times 20^3}{3075 \times \frac{1}{2}} = 62440$. The fourth root of this number is 15·8 inches, the depth required.

The deflection of $\frac{1}{40}$ of an inch for each foot in length is not injurious to ceilings; indeed, the usual allowance for settlement is about twice that quantity. Ceilings have been found to settle about 4 times as much without causing cracks, and have been raised back again without injury.

The variable load on a floor seldom can exceed half the quantity of 120 lbs. on a superficial foot, unless it be in public rooms; hence, the number may be taken from 60 to 120, according to circumstances.

The same rule applies to joists of different kinds for floors; the area of the floor supported by the joists being multiplied by from 60 to 120 lbs. per superficial foot, according to the use the room is designed for.

Example III.—To determine the size of a rafter for a roof to support the covering of slate, the distance between the supports being 6 feet, and the weight of a superficial foot, including the stress of the wind, being 56 lbs., and the deflection not to exceed $\frac{1}{40}$th of an inch for each foot in length.

The tabular value of E is 3075, the weight $= 56 \times 6 = 336$ lbs., and (by the rule and note 4,) we have $\frac{5 \times 336 \times 40 \times 6^3}{8 \times 3075 \times 6} = 98{\cdot}34$. Now, if the breadth be made $2\frac{1}{2}$ inches, we have $\frac{98{\cdot}34}{2{\cdot}5} = 39{\cdot}3$; and the cube root of 39·3 is 3·4 inches, the depth required.

PROBLEM VII.

To determine the Dimensions of a Pillar or Column to bear a given Stress in the direction of its Axis, without sensible Curvature.

Rule.—Multiply the weight to be supported in lbs. by the square of the length of the pillar in feet, and divide the product by 2·5 times the tabular value of E, (p. 231,) the quotient will be equal to the breadth multiplied by the cube of the least thickness; therefore, either the breadth or thickness will require to be fixed upon, before the other can be found.*

* The rule is derived as follows:—the force f, which a column will bear without sensible flexure is $f = {\cdot}8225 \frac{d^2 m}{l^2}$; and $m = \frac{l^3 W}{4 d^2 \delta}$

Note 1.—If the pillar be square, its side will be the fourth root of the quotient.

Note 2.—If the column be a cylinder, multiply the tabular value of E by 1·5 instead of 2·5. The fourth root of the quotient in the rule will be the diameter of the cylinder.

Example I.—What should be the least thickness of a pillar of oak to bear a ton without sensible flexure, its breadth being 3 inches, and its length 5 feet?

The tabular value of E for oak is 3360, and one ton = 2240 lbs.; hence, $\frac{2240 \times 5^2}{2{\cdot}5 \times 3360 \times 3} = 2{\cdot}222$. The cube root of 2·222 is 1·31 nearly, which is the side as required.

Example II.—Required the side of a square post of Riga fir to support 10 tons, the pressure being in the direction of the axis, and the height of the post 12 feet.

The tabular value of E is 3075, hence, $\frac{22400 \times 12^2}{2{\cdot}5 \times 3075} = 419{\cdot}6$ nearly; the fourth root of which is 4·53 inches, the side of the post as required.

The dimensions given by this rule are obviously too small to be used in practice. The rule only shews the extreme load that can be supported by a pillar under the theoretical condition that the pres-

(see Dr. Young's Nat. Phil. II. pp. 47, 48); hence, when l is in feet, we have $f = \frac{2{\cdot}4675\, l\, W}{\delta}$. But we have $W = \frac{E\, a\, d^3\, \delta}{l^3}$; consequently, $f = \frac{2{\cdot}4675\, E\, a\, d^3}{l^2}$. In the rule in words 2·5 is used for 2·4675. If the above expression be divided by 1·7, it becomes a rule for a cylinder, or $\frac{1{\cdot}4508\, E\, d^4}{l^2} = f$, or $\frac{1{\cdot}5\, E\, d^4}{l^2} = f$, for simplicity.

sure exactly coincides with the axis of the pillar; and it is shewn that this pressure will overpower the resistance of the pillar if it has the smallest deviation from the axis. (See Dr. Young's Nat. Phil. II. p. 47). It is the more necessary to point out this circumstance, because it is the same in Girard's rules, quoted in p. 75; and Poisson's Equation, (Traité de Méchanique, Art. 160, Tome I). For the case where the force is applied at a distance from the axis, Poisson has left the solution incomplete. Dr. Young has given a solution of this case in his work above quoted; but it is not quite so convenient for application as one which may be obtained by assuming certain data, that are difficult to obtain in a simple form by calculation.

PROBLEM VIII.

To determine the Stress a Pillar or Column will bear when the Direction of the Straining Force does not coincide with the Axis of the Support.

Rule.—Multiply the tabular value of C by the breadth and square of the depth, both in inches.

Divide 6 times the square of the length in inches, by the depth multiplied into the tabular value of U; and add to the quotient the depth, and also 6 times the distance in inches, of the direction of the force from the axis.

Divide the first product by this sum, and the resulting quotient will be the greatest load the pillar or column will bear.*

* By Problem II. the load a pillar will bear is $\frac{C\,a\,d^2}{d+6\,c}$ when c is

Note 1.—If the force coincides with the axis, the depth only is to be added to the quotient in the second part of the rule.

Note 2.—In practice, one-fourth of the stress should be the utmost allowed.

Example I.—Required the greatest weight a post of Riga fir would support, its length being 12 feet, or 144 inches, and its side 5 inches; and the distance of the direction of the pressure from the axis, 2 inches.

In this case, C = 10707, and U = 588, according to the Table of Data.

$$\text{Therefore, } \frac{10707 \times 5 \times 5^2}{\frac{6 \times 144^2}{5 \times 588} + \overline{6 \times 2} + 5} = 22570 \text{ lbs.}$$

If the distance of the direction of the force from the axis be omitted, we have

$$\frac{10707 \times 5^3}{\frac{6 \times 144^2}{5 \times 588} + 5} = 28300 \text{ lbs.*}$$

In either case, divide the weight given by the rule by 4, for the greatest load to be allowed in practice.

Example II.—A curved rib of oak, 10 feet in length, has to sustain a pressure in the direction of its chord line; the greatest distance of its axis from

the greatest deviation of the direction from the axis; but when a column exceeds a certain length, it bends, and, to find its ultimate strength, this flexure should be added to the deviation from the axis. Assume, that the flexure is equal to the ultimate deflection of a beam supported at the ends and loaded in the middle, which is $\Delta = \frac{l^2}{d\,U}$, we shall have $\frac{C\,a\,d^2}{d + 6c + \frac{6\,l^2}{d\,U}}$ = the greatest weight the pillar will bear; and this is the same as the rule.

* If the former rule be employed, it gives 33365 lbs.

the chord line is 20 inches, its depth is 4 inches, and breadth 6 inches; required the pressure it will sustain?

For oak, the tabular value of C is 10853, and of U it is 435; hence, $\dfrac{10853 \times 6 \times 4^2}{\dfrac{6 \times 120^2}{4 \times 435} + \overline{6 \times 20} + 4} = 6000$ lbs. nearly; and in practice it may be loaded with $\frac{6000}{4} = 1500$ lbs.

COMPARISON OF EXPERIMENTS.

154. The preceding rules apply to practical questions, and, being reduced to the simplest form they admit of, they are not adapted for comparing the results of experiments; and, therefore, the addition of the following problem seemed necessary.

PROBLEM I.

To compute the ultimate Deflection of Beams, or Rods, before their Rupture.

Note.—The beams are supposed to be supported at each end.

Rule.—Multiply the tabular value of U, in the preceding Table of Data, by the depth of the beam in inches, and divide the square of the length, also in inches, by that product, for the ultimate deflection sought. (Art. 103—7.)

Example I.—A square inch rod of ash, 6 feet long, is broken by a weight applied to its centre:—how much will it be deflected before it breaks?

395 = tab. value of U,
1 depth,

395 first product.

6 feet = 72 inches, and $72^2 = 5184$.

395) 5184 (13·1 = $13\frac{1}{10}$ inches nearly.
395

1234
1185

490
395

95

Example II.—What will be the ultimate deflection of a similar rod, 12 feet long?

Here, 12 feet = 144 inches, and $144^2 = 20736$.

395) 20736 (52·5, or 4 feet $4\frac{1}{2}$ inches nearly.
1975

986
790

1960
1975

Example III.—A half-inch plank of larch, similar to our 3d specimen, being 10 feet in length, how many inches will the centre descend before it fractures?

518 tab. value of U,
·5 = $\frac{1}{2}$ inch depth,

259·0

10 feet = 120 inches, and $120^2 = 14400$.

259) 14400 (55·5 inches, or 4 feet $7\frac{1}{2}$ inches,
1295

1450
1295

1550
1295

255

Note 1.—The same rule applies to beams, or rods, *fixed* at both ends.

Note 2.—It is obvious that our deflection, according to this rule, may exceed half the length of the piece. When this happens, it indicates that the two ends may be brought in contact without the rupture taking place.

Note 3.—When the piece is fixed at one end only, the deflection will be 8 times what is given by the above rule.

PROBLEM II.

To find the ultimate Transverse Strength of any Rectangular Beam of Timber, fixed at one End and loaded at the other.

Rule.—1. Compute the ultimate deflection by note 3 of the last problem, and divide the deflection by the length, which will give the sine of the angle of deflection; whence, by a table, find the secant.

2. Multiply this secant by the breadth and square of the depth in inches, and the product again by the value of $\acute{S}$ in the Table of Data.

3. Divide this last product by the length in inches, and the quotient will be the answer, in lbs. (Art. 115—1.)

Example I.—What weight will it require to break a piece of Mar forest fir, fixed by one end in a wall, and loaded at the other; the breadth being 2 inches, depth 3 inches, and length 4 feet?

First, to find the ultimate deflection, we have

$$\frac{48^2}{3 \times 403} \times 8 = \frac{18432}{1209} = 15 \text{ inches.}$$

$$\frac{15}{48} = \cdot 3125 = \text{sin. } 18^\circ\ 13';$$

$$
\begin{array}{r}
1{\cdot}0527 = \text{sec. } 18^\circ\ 13' \\
18 = 2 \times 3^2 \\
\hline
84216 \\
10527 \\
\hline
18{\cdot}9486 \\
1310 = S' \\
\hline
1894860 \\
568458 \\
189486 \\
\hline
6)\,24822{\cdot}6660 \\
\hline
4 \text{ feet} = 48 = 8)\ \ 4137\phantom{{\cdot}6660} \\
\hline
518 \text{ lbs., } \textit{answer}.
\end{array}
$$

Example II. A square oaken balk, 12 inches square, projects 8 feet 4 inches from a solid wall, in which it is fixed; what weight will be sufficient to break it?

First, to find the ultimate deflection, we have

$$\frac{100^2}{435 \times 12} \times 8 = 15{\cdot}325;$$

$$\frac{15{\cdot}325}{100} = {\cdot}15325 = \text{sin. } 8^\circ\ 49'.$$

$$
\begin{array}{r}
1{\cdot}0119 = \text{sec. } 8^\circ\ 49', \\
1728 = 12 \times 12^2 \\
\hline
80952 \\
20238 \\
70833 \\
10119 \\
\hline
1748{\cdot}5632 \\
1736 \qquad = S' \\
\hline
10488 \\
5244 \\
12236 \\
1748 \\
\hline
1{,}00)\ 30345{,}28 \\
\hline
30345 \text{ lbs., } \textit{answer}.
\end{array}
$$

Example III. A piece of ash, 2 inches square, projects 6 feet from a wall in which it is fixed; what weight, uniformly distributed through its length, will be required to break it? *Ans.* 450 lbs.

Example IV. A piece of Mar forest fir, fixed by one end in a wall, and projecting 4 feet, was broken by a weight of 1000 lbs. suspended at its extremity. Now, supposing the beam to be square, and similar to our 3d specimen, Table, page 202, what was the area of its transverse section? *Ans.* 11·31 square inches.

Note 1.—When the beam is loaded uniformly throughout its length, the same rule will still apply, except that the last result must be doubled.

PROBLEM III.

To compute the ultimate Transverse Strength of any Rectangular Beam, when supported at both Ends and loaded in the Centre.

Rule.—1. Compute the ultimate deflection by Prob. II.; square that deflection, and divide it by the square of half the length of the beam, and add the quotient to 1, for the square of the secant of deflection; which multiply by the length in inches.

2. Multiply the tabular value of $\acute{S}$ by 4 times the breadth, and the square of the depth; and divide that product by the former, for the answer in lbs. (Art. 115—2.)

Example I.—What weight will be necessary to

break a piece of larch similar to our 3d specimen, the length being 8 feet 4 inches, the breadth 8 inches, and depth 10 inches; being supported at each end, and loaded in the middle?

First, to find the ultimate deflection, we have

$$\frac{100^2}{403\times 10}=\frac{10000}{4030}=2\cdot5 \text{ inches nearly.}$$

$$\frac{2\cdot5^2}{50^2}+1=\frac{\cdot01}{4}+1=1\cdot0025=\text{sq. sec. def.}$$

```
            1310 = tab. value of S',
            3200
           ------
          262000
          3930
         -------
100·25) 4192000 (41815 lbs. answer by 2d rule.
        40100
         -----
          18200
          10025
          -----
           81750
           80200
           -----
            15500
            10025
            -----
             54750
```

Note 1.—When the beam is loaded uniformly throughout its length, the same rule will apply, but the result must be doubled.

2.—If the beam be *fixed* at each end and loaded in the middle, then the result obtained in the problem must be increased by its half.

3.—If the beam be fixed at both ends and loaded uniformly throughout its length, the same result must be multiplied by 3. (Art. 115.) That is, the breaking weights, under these several circumstances, are,

Supported and loaded in the centre....	are as	1
Do. and loaded throughout its length..		2
Fixed and loaded in the centre		$1\frac{1}{2}$
Do. loaded throughout its length		3

Example II. A piece of New England fir, 10 feet long, and 6 inches square, being fixed at each end, and loaded uniformly through its entire length: it is required to find the weight necessary to break it.

By the Second Rule.

First, to find the ultimate deflection, we have

$$\frac{120^2}{757 \times 6} = 3\cdot2 \text{ inches nearly, deflection.}$$

$$1 + \frac{3\cdot2^2}{60^2} = 1 + \frac{10\cdot24}{3600} = 1\cdot0028 = \text{sq. of sec. of def.}$$

```
1·0028
   120 length,
────────
120·3360 first product.

  1116 tab. value of S'
   864 = 4 × 6 × 6²
  ────
  4464
 6696
8928
  ────
120·336)964224 (8012 lbs.
        962688
        ──────
          153600
          120336
          ──────
           332640
           240672
           ──────
            91968
```

That is, the beam will require 8012 lbs. to break it, when supported and loaded in the middle.

```
Hence,   8012
            3
         ────
24036 lbs., the answer.
```

Example III.—Required the strength of a joist of Riga fir, 1st specimen, the bearing 10 feet, and lateral dimensions 8 inches by 3 inches; the weight being equally distributed. *Ans.* 14182 lbs.

Example IV.—Required the same of a girder, uniformly loaded, having 16 feet bearing, and lateral dimensions 14 inches by 10 inches. *Ans.* 90486 lbs.

PROBLEM IV.

*To compute the Transverse Strength of a Beam of any Form and Dimensions; the Strength of direct Cohesion and the Ratio of Tension and Compression being given.**

Rule 1.—Divide the given area, or section, into two such parts, that the product of the area of tension into the distance of its centre of gravity may be to the area of compression into the distance of its centre of gravity in the given ratio.†

2. Find the strength of direct cohesion of the area of tension, and multiply it by the distance of the centre of gravity from the neutral axis: then divide

* That is, the ratio of the area of tension into the distance of its centre of gravity from the neutral axis, to that of the area of compression into the distance of its centre of gravity from the same line, which, we have seen, is as 3 to 1 in fir.

† By comparing this rule with the theorem given by Guldin, commonly called the *centro-baryc method*, it will be found to be the same as saying, that the area or section must be divided into two such parts, that the solid of revolution of the respective parts about the dividing line may have to each other the given ratio.

this product by half the length of the beam, and the quotient will be the answer, or the weight necessary to break the beam.

Note 1.—This rule supposes the beam fixed at one end, and loaded at the other; if it be supported at both ends, and loaded in the middle, proceed in the same manner; but, in this case, the result obtained as above must be multiplied by 4; and if the beam be fixed at both ends, by 6.

2.—If in any of these cases the weight be uniformly distributed, then the above results must be doubled.

Example I.—Required the strength of, or the weight requisite to break, an equilateral triangular piece of fir; supposing the strength of direct cohesion to be 10,000 lbs., the ratio of the two forces as 3 to 1, and the beam to be fixed in a wall, with its base downwards; the side of the triangle 4 inches, and length 8 feet.

Let A B C, *fig*. 8, *plate* III., denote the section of the beam, and G its centre of gravity; E C F, the area of tension; g', its centre of gravity; A E F B, the area of compression; and its centre of gravity, g: then we must find

$$\mathrm{A\,E\,F\,B} \times n\,g : \mathrm{E\,C\,F} \times n\,g' : 3 : 1;$$

which, by a well-known property of the centre of gravity, is the same as

$$\mathrm{E\,C\,F} \times n\,g : \mathrm{A\,B\,C} \times n\,\mathrm{G} : 1 : 2.$$

Let A B $= b$, D C $= a$, and n C $= x$, then,

$$a : b :: x : \frac{b\,x}{a} = \mathrm{E\,F}, \text{ and}$$

$$\frac{b\,x}{a} \times \frac{x}{2} = \frac{b\,x^2}{2\,a} = \text{area E C F},$$

$$\frac{b\,a}{2} = \text{area A B C}.$$

Also, C G $= \frac{2}{3} a$, n G $= \frac{2}{3} a - x$, and $n g' = \frac{1}{3} x$. Hence, then,

$$\frac{b x^2}{2 a} \times \frac{1}{3} x : \frac{b a}{2} \times (\tfrac{2}{3} a - x) : : 1 : 2, \text{ or}$$

$$\frac{b x^3}{3 a} = -\frac{b a x}{2} + \frac{b a^2}{3}.$$

Which reduces to

$$x^3 + \tfrac{3}{2} a^2 x = a^3.$$

In the present case, $a = \sqrt{12} = 2\sqrt{3} = 3{\cdot}464$; and the equation in numbers becomes

$$x^3 + 18 x = 41{\cdot}568.$$

Whence we find $x = 1{\cdot}916$ nearly, and

$\frac{b x^2}{2 a} = \frac{4 \times 1{\cdot}916^2}{6{\cdot}928} = 2{\cdot}12$ area E C F nearly.

Again, $2{\cdot}12 \times \frac{1{\cdot}916}{3} \times 10000 = 13540$ nearly.

Therefore, $\frac{13540}{48} = 282$ lbs., *answer*.

In a similar manner the strength of any other formed beam may be ascertained.

Example II.—Required the strength of the same beam, with the vertex downwards?

Example III.—With the same data as those above, it is required to compare the strength of a square beam in the two following cases; viz. with its diagonal vertical, and with its side vertical.

Example IV.—With the same data, find the weight necessary to break a cylinder 4 feet long and 4 inches in diameter.

MISCELLANEOUS PROBLEMS.

1. Required the weight necessary to tear asunder a king-post of oak, which is 8 inches by 6 inches?

2. A piece of Riga fir, having 24 feet bearing, 7 inches deep and 6 inches thick, is loaded uniformly throughout its length; what weight will be required to break it?

3. What weight will it require to break the same, supposing it to be suspended from a point which is 18 feet from one end and 6 from the other? See Art. 108.

4. A rafter of the same wood, 24 feet long, and 9 inches deep by 7 in breadth at one end, and 7 by 6 at the other, being placed horizontally, and loaded at its middle point; what weight will be required to break it, and in what part will the fracture take place?

5. What weight will the same rafter bear when placed at an angle of 45°, as in a roof of a true pitch, supposing the weight to be distributed throughout its length?

6. What would be the difference in the strength of two fir girders of 24 feet length; the one being 14 inches by 10 inches, and the other 15½ inches by 9 inches; and what would be their proportional resistance to deflection?

7. What is the greatest weight that may be drawn up, or let down, on an oak plank, 10 inches in breadth, 2 inches in thickness, and 12 feet in length, without breaking it, when inclined at an angle of 20°?

8. A tree, from the forest of Mar, is 30 feet in length, and 24 inches in diameter at its upper end: required the dimensions of the strongest rectangular beam that can be cut out of it, and what weight will be sufficient to break it, supposing it to be loaded at its centre, and its two ends resting upon two props, with its greatest lateral dimension vertical, and of a quality similar to our third specimen?

Note.—The first part of this question is the same as requiring the dimensions of a rectangle inscribed in a given circle, such, that the breadth into the square of the length shall be a maximum.

APPENDIX.

EXPERIMENTS

ON THE

DIRECT AND TRANSVERSE RESISTANCE OF IRON WIRE

OF DIFFERENT LENGTHS AND DIMENSIONS.

By T. TELFORD, Esq. F.R.S.E. &c.

The following course of experiments, which I owe to the liberal views of the above gentleman, was made with a particular object; viz. the obtaining of data connected with the erection of an iron hanging bridge over the river Mersey, at Runcorn, near Liverpool; which, from the nature of the navigation, was to consist of only three spans, or openings; the centre one of 1000 feet, and the two others 500 feet each; making the entire length 2000 feet. It was also necessary to keep the intrados 70 feet above high-water mark. An arch bridge, under these circumstances, is conceived to be wholly impracticable: and both courage and genius were requisite to conceive any practicable construction. Mr. Telford proposed, however, an iron hanging bridge, to consist of 16 cables or bars, each formed of 36 square $\frac{1}{2}$ inch iron bars; and the segments of cylinders, proper for forming them into one immense cylindrical iron cable, which, in its whole length, including the fixings on shore, would be nearly half a mile, and about $4\frac{1}{4}$ inches diameter; that is, the diagonal of the square section of the 36 half inch bars will be equal to $\sqrt{18} = 4{\cdot}24$ inches; and this diagonal will obviously be the same as the diameter of the cylinder, after the segments above mentioned are applied to the four sides of the square prism.

Each of these ½ inch bars, as well as the four segments, were to have been welded into one length; and being well secured with bucklings at every 5 feet, and wrapped in flannel well saturated with a composition of rosin and bees'-wax, to preserve them from the weather, they were to have been farther bound together with wire of about $\frac{1}{10}$th inch diameter, forming, as we have said above, one immense iron cable of the entire length; and from 16 of these cables, the road ways, which were to consist of two for the passing and repassing of carriages, and a centre one for foot passengers, were to be suspended. The two principal supports for forming the centre span were intended to be about 140 feet in height; and the deflection of the centre of the inverted arch, or catenary, $\frac{1}{20}$th of the opening; viz. 50 feet: and the two side spans to consist of two semi-catenaries, which were originally designed to be of the same curvature as the principal centre one; viz. the lowest point of the centre catenary was proposed to be exactly in the same horizontal line as the two extremities of the side openings, which would have effectually relieved the two principal supports from any horizontal draw, or of any tendency to overturn: but some slight modification of this plan was afterwards made, for the sake of reducing the expense, which would bring the extremities of the side semi-catenaries lower than the lowest point of the centre one.

An undertaking of such an immense magnitude, so perfectly original, and which, when completed*, will perhaps

* At the time the above was written, viz. previous to the publication of the first edition of this work, the bridge here described was in contemplation, and some steps had been taken towards obtaining funds for its erection; but I am not aware that any thing has yet been done in reference to its actual construction. Mr. Telford, however, has at this time just completed a bridge, on the same principles, over the Menai Strait; but the span does not exceed much more than half that proposed for the Mersey. And Captain Brown has already constructed one, but of still less span, over the Tweed. The idea, however, of employing continuous cables, such as those above described, has been relinquished, and a succession of bars, jointed together, substituted in their place. By this means it is conceived that any link may be repaired and replaced without destroying the effect of the line of suspension of which it forms a part.

be one of the most singular works of art that any age or nation ever produced, ought not to be attempted without the best data that could possibly be obtained, relative to the strength of the proposed materials, under all the variety of strains to which they are likely to be exposed: and the following experiments were therefore made, as before stated, with this particular view.

In order to comprehend the tabulated results, it will be necessary to explain the apparatus with which the experiments were made: these are presented in *fig*. 9, *plate* III.

Here R S, T V, represent the supporting pillars upon which the wire was extended; Q S, another prop over which the wire passed; being placed at such an angle as made it coincide with the direction of the resultant of the vertical and horizontal tensions, in order to prevent any strains upon the other support, R S.

A, B, C, D, represent the places of the several weights with which the wire was loaded; C being in the centre of the length, and B and D at ¼th of the length from each end; and the deflections from the horizontal line R T were measured at these points, as the different weights were applied.

EXPERIMENT, No. 1.

Distance of the Props, 100 *feet; Weight of* 100 *feet of Wire,* $29\frac{1}{2}$ *ounces; Diameter, rather more than* $\frac{6}{70}$*ths of an inch; and it broke when suspended vertically, at a Medium of different Trials, with* 531 *lbs.*

Weight at A, including the Wire Q A.		*Weight at B.*		*Weight at C.*		*Weight at D.*		*Deflection at B.*		*Deflection at C.*		*Deflection at D.*		*REMARKS.*
lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	ft.	in.	ft.	in.	ft.	in.	
5	$6\frac{1}{2}$	0	0	0	0	0	0	—		4	10	—		Deflections at B and D not taken.
10	5	0	0	0	0	0	0	—		2	$11\frac{1}{2}$	—		
30	$5\frac{1}{5}$	0	0	0	0	0	0	—		0	$10\frac{1}{2}$	—		
do.		0	0	1	$0\frac{1}{2}$	0	0	—		1	8	—		
do.		0	0	2	$0\frac{1}{2}$	0	0	—		2	7	—		
do.		0	0	5	$0\frac{1}{2}$	0	0	—		4	11	—		The Weight at C being taken off, the Deflection became 11 in.
176	0	5	0	30	4	5	0	2	1	4	$6\frac{1}{2}$	2	1	
do.		9	0	30	4	5	0	2	$5\frac{1}{2}$	4	$10\frac{1}{4}$	2	$2\frac{1}{2}$	Raised weight A 1 in.
226	0	9	0	56	0	5	0	3	11	7	$10\frac{1}{2}$	3	$7\frac{1}{2}$	
286	0	9	0	56	0	5	0	2	$8\frac{3}{4}$	5	$11\frac{1}{2}$	2	$6\frac{1}{2}$	
342	0	9	0	56	0	5	0	2	$3\frac{1}{2}$	5	$0\frac{3}{4}$	2	$1\frac{3}{4}$	
do.		9	0	66	0	5	0	2	5	5	$4\frac{1}{2}$	2	$3\frac{1}{4}$	
do.		9	0	72	0	5	0	2	7	5	$9\frac{1}{2}$	2	$5\frac{1}{4}$	
do.		9	0	77	0	5	0	2	7	5	10	2	$5\frac{1}{4}$	
do.		9	0	81	0	5	0	2	$9\frac{3}{4}$	6	$4\frac{3}{4}$	2	8	
do.		9	0	87	0	5	0	2	$10\frac{1}{4}$	6	$6\frac{1}{4}$	2	$8\frac{1}{2}$	
do.		15	0	71	0	15	0	2	$11\frac{3}{4}$	6	$3\frac{3}{4}$	2	$11\frac{3}{4}$	
402	0	15	0	71	0	15	0	2	$8\frac{1}{4}$	5	$8\frac{3}{4}$	2	$8\frac{1}{4}$	
402	0	30	0	56	0	30	0	—		—		—		Broke after sustaining these Weights for a short time.

EXPERIMENT, No. 2.

Distance of the Props, 31 *ft.* 6 *in.; the same specimen of Wire as in Experiment No.* 1, *but had not been before used; the two Ends of the Wire, in this Experiment, were fixed, after drawing it as tight as possible; viz. to within less than* $\frac{1}{8}$*th of an inch of a horizontal Line; and the Weights applied only in the centre.*

End at R and T fixed.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.	Deflection at C.	Deflection at D.	REMARKS.
		lbs.			ft. in.		
Fixed	0	$10\frac{1}{4}$	0	—	0 2·83	—	
do.	0	$20\frac{1}{4}$	0	—	0 5·5	—	
do.	0	$30\frac{1}{4}$	0	—	0 7·75	—	
do.	0	$40\frac{1}{4}$	0	—	0 10	—	
do.	0	$50\frac{1}{4}$	0	—	1 0	—	
do.	0	$60\frac{1}{4}$	0	—	1 1·75	—	
do.	0	$70\frac{1}{4}$	0	—	1 3·5	—	
do.	0	$80\frac{1}{4}$	0	—	1 5	—	
do.	0	$90\frac{1}{4}$	0	—	1 6·5	—	
do.	0	$100\frac{1}{4}$	0	—	1 8	—	
do.	0	$110\frac{1}{4}$	0	—	1 9·75	—	
do.	0	$120\frac{1}{4}$	0	—	1 10·75	—	
do.	0	$130\frac{1}{2}$	0	—	—	—	

Just bore the last weight, and then broke.

EXPERIMENT, No. 3.

Distance of Props, 100 *ft.; Diameter,* $\frac{1}{10}$*th of an inch; Weight of* 100 *feet*=2 *lb.* 9 *oz.: bore vertically* 736 *lbs., but broke with* 738 *lbs.*

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.	Deflection at C.	Deflection at D.	REMARKS.
lbs.	lbs.	lbs.	lbs.	ft. in.	ft. in.	ft. in.	
362	0	0	0	—	0 5	—	
362	30	15	30	2 2	2 $11\frac{3}{4}$	2 $1\frac{1}{4}$	
362	35	30	35	2 8	3 $10\frac{3}{8}$	2 $7\frac{1}{4}$	
362	40	35	40	2 $11\frac{4}{10}$	4 $3\frac{1}{2}$	2 $10\frac{1}{2}$	
362	40	41	40	3 3	4 11	3 $2\frac{1}{4}$	
468	56	41	56	3 $4\frac{6}{10}$	4 $9\frac{4}{10}$	3 $4\frac{7}{10}$	
498	56	41	56	3 $0\frac{4}{10}$	4 $3\frac{6}{10}$	3 $0\frac{6}{10}$	
558	61	41	61	3 $1\frac{1}{2}$	4 $4\frac{1}{4}$	3 $1\frac{1}{4}$	
608	76	76	76	3 $5\frac{9}{10}$	5 $3\frac{3}{10}$	3 $6\frac{1}{2}$	Fixed the wire at A.
Fixed	56	56	56	3 0	4 $6\frac{7}{10}$	2 $11\frac{1}{2}$	
do.	71	68	71	3 $3\frac{8}{10}$	5 0	3 4	Refixed the wire.
do.	do.	do.	do.	3 $4\frac{7}{10}$	5 $1\frac{3}{10}$	3 $4\frac{7}{10}$	
do.	77	74	77	3 $6\frac{2}{10}$	5 $4\frac{8}{10}$	3 $6\frac{8}{10}$	Refixed the wire.
do.	77	74	77	3 $3\frac{7}{10}$	4 $11\frac{8}{10}$	3 $3\frac{2}{10}$	

Bore this weight; but in attempting to add 4 pounds more to the weights at B and D, the wire broke.

EXPERIMENT, No. 4.

The same Wire as in last Experiment. Distance of the Props, 31 *ft.* 6 *in.*

Weight at A.	*Weight at B.*	*Weight at C.*	*Weight at D.*	*Deflection at B.*		*Deflection at C.*		*Deflection at D.*		*REMARKS.*
	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
Fixed	0	0	0	—		0	$0\frac{1}{8}$	—		Both ends fixed.
do.	40	41	40	0	$7\frac{5}{8}$	0	$10\frac{7}{8}$	0	$7\frac{1}{2}$	
do.	44	47	44	0	$8\frac{1}{2}$	1	$0\frac{1}{8}$	0	$8\frac{1}{2}$	
do.	50	47	50	0	9	1	$0\frac{5}{8}$	0	9	
do.	56	47	56	0	$9\frac{3}{4}$	1	$1\frac{1}{4}$	0	$9\frac{1}{2}$	
do.	56	53	56	0	$10\frac{1}{8}$	1	2	0	$9\frac{3}{4}$	
do.	61	53	61	0	$10\frac{1}{2}$	1	$2\frac{3}{8}$	0	$10\frac{1}{4}$	
do.	61	59	61	0	$10\frac{3}{4}$	1	$3\frac{1}{8}$	0	$10\frac{3}{4}$	
do.	67	68	67	1	0	1	$4\frac{5}{8}$	0	$11\frac{5}{8}$	
do.	71	68	71	1	0	1	$4\frac{7}{8}$	1	0	
do.	71	76	71	1	$0\frac{1}{2}$	1	$5\frac{1}{8}$	1	$0\frac{1}{4}$	

With the last weights suspended a few minutes, the wire broke.

EXPERIMENT, No. 5.

Distance of the Props, 100 *feet: Diameter,* $\frac{6}{100}$ *of an inch; Weight of* 100 *feet,* $16\frac{1}{2}$ *ounces. Vertically, the Wire bore* 277 *lbs. a few minutes, and then broke.*

Weight at A.	*Weight at B.*	*Weight at C.*	*Weight at D.*	*Deflection at B.*		*Deflection at C.*		*Deflection at D.*		*REMARKS.*
lbs.	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
180	0	0	0	0	$1\frac{3}{4}$	0	$1\frac{1}{4}$	0	$1\frac{3}{4}$	
180	6	5	6	1	$0\frac{7}{8}$	1	$5\frac{1}{2}$	0	$11\frac{3}{4}$	
180	12	10	12	1	$10\frac{1}{8}$	2	$7\frac{3}{4}$	1	$9\frac{1}{2}$	
210	16	14	16	2	$3\frac{1}{2}$	3	$2\frac{1}{2}$	2	2	
248	16	14	16	2	$2\frac{5}{8}$	3	$2\frac{1}{2}$	2	$2\frac{1}{4}$	Took off the weight A, and tightened the wire.
Fixed	16	14	16	1	$9\frac{5}{8}$	2	$7\frac{1}{4}$	1	$9\frac{1}{4}$	Broke the wire in attempting to draw it tighter.
Another piece of the same Wire.										
Fixed	0	0	0	0	$2\frac{3}{4}$	0	4	0	$3\frac{1}{4}$	
do.	16	15	16	2	4	3	5	2	$4\frac{7}{8}$	
do.	22	19	22	2	$7\frac{1}{2}$	3	10	2	$8\frac{1}{10}$	

In attempting to increase these weights to 25, 26, and 27lbs., the wire broke at a defective place.

EXPERIMENT, No. 6.

Same Wire as in the preceding Experiment. Distance of the Props, 31 *feet* 6 *inches.*

Weight at A.	*Weight at B.*	*Weight at C.*	*Weight at D.*	*Deflection at B.*		*Deflection at C.*		*Deflection at D.*		*REMARKS.*
	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
Fixed	22	30	22	0	$11\frac{1}{2}$	1	6	0	$10\frac{7}{8}$	
do.	28	30	28	1	$1\frac{1}{4}$	1	$6\frac{1}{2}$	1	$0\frac{5}{8}$	
do.	30	30	30	1	$1\frac{5}{8}$	1	$6\frac{1}{2}$	1	$1\frac{1}{8}$	
do.	30	35	30	1	$1\frac{1}{2}$	1	$7\frac{5}{8}$	1	$1\frac{1}{8}$	

Broke in attempting to add 4lb. more at B and D.

EXPERIMENT, No. 7.

Distance of the Props, 140 *feet: Diameter,* $\frac{1}{21}$ *of an inch: Weight of* 140 *feet,* 14 *ounces. Broke, vertically, with* 157 *lbs.*

Weight at A.	*Weight at B.*	*Weight at C.*	*Weight at D.*	*Deflection at B.*		*Deflection at C.*		*Deflection at D.*		*REMARKS.*
lbs.	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
120	0	0	0	0	$1\frac{1}{2}$	0	$1\frac{5}{8}$	0	$1\frac{3}{8}$	
120	6	5	6	2	8	3	$5\frac{2}{10}$	2	$7\frac{3}{4}$	
120	12	10	12	4	$8\frac{3}{10}$	6	$4\frac{1}{2}$	4	$7\frac{7}{10}$	
120	15	20	15	7	$1\frac{1}{2}$	10	0	7	$0\frac{3}{4}$	
132	15	20	15	6	$3\frac{1}{4}$	8	$9\frac{1}{2}$	6	$4\frac{1}{2}$	
132	21	25	21	8	$8\frac{1}{2}$	11	11	8	7	
150	21	25	21	7	$11\frac{1}{2}$	10	10	7	0	
150	25	25	25	8	3	10	11	8	2	Broke.

EXPERIMENT, No. 8.

Same Wire as in the last Experiment. Distance of the Props, 31 *ft.* 6 *in.*

Weight at A.	*Weight at B.*	*Weight at C.*	*Weight at D.*	*Deflection at B.*		*Deflection at C.*		*Deflection at D.*		*REMARKS.*
	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
Fixed	0	0	0	0	$5\frac{1}{2}$	0	$5\frac{1}{4}$	0	$4\frac{1}{2}$	
do.	6	5	6	1	$1\frac{3}{4}$	1	$4\frac{1}{4}$	1	$1\frac{1}{2}$	
do.	12	10	12	1	$4\frac{3}{8}$	1	8	1	$3\frac{1}{4}$	
do.	16	15	16	1	$6\frac{1}{4}$	1	$10\frac{1}{2}$	1	$4\frac{7}{8}$	
do.	20	20	20	1	$7\frac{1}{2}$	2	1	1	$6\frac{3}{8}$	

Broke in attempting to add 2 lbs. at B, 4 lbs. at C, and 2 lbs. at D.

EXPERIMENT, No. 9.

The same Wire as last Experiment, and the Props the same distance; viz. 31 *ft.* 6*in.*

Weight at A.	*Weight at B.*	*Weight at C.*	*Weight at D.*	*Deflection at B.*		*Deflection at C.*		*Deflection at D.*		*REMARKS.*
lbs.	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
120	20	30	20	2	6	3	$3\frac{1}{2}$	2	$2\frac{1}{4}$	
120	25	30	20	2	$9\frac{1}{4}$	3	7	2	5	
120	31	34	31	3	$5\frac{4}{10}$	4	$4\frac{1}{2}$	2	$11\frac{1}{2}$	
120	34	34	34	3	$6\frac{3}{4}$	4	$5\frac{1}{2}$	3	$1\frac{1}{4}$	
120	34	42	34	3	$9\frac{3}{4}$	4	$11\frac{1}{4}$	3	$2\frac{3}{4}$	
120	34	50	34	4	0	5	$3\frac{1}{4}$	3	4	
150	34	50	34	3	$3\frac{6}{10}$	4	$4\frac{1}{2}$	2	$9\frac{8}{10}$	
150	34	55	34	3	$6\frac{1}{4}$	4	$8\frac{1}{2}$	3	0	
150	37	55	37	3	$9\frac{4}{10}$	5	0	3	$2\frac{1}{2}$	
150	37	56	37	3	$9\frac{1}{4}$	5	0	3	$2\frac{1}{2}$	
156	37	56	37	3	$9\frac{1}{4}$	5	0	3	$2\frac{1}{2}$	
160	39	57	39	3	$9\frac{2}{10}$	5	$0\frac{3}{10}$	3	$2\frac{2}{10}$	Broke in attempting to add 6 lbs. more.

Note.—The above Experiments were made at the patent iron cable manufactory of Messrs. Brunton and Co.

EXPERIMENT, No. 10.

Distance of the Props 900 *feet. Diameter of Wire,* $\frac{1}{10}$ *inch; Weight of* 900 *feet,* 28 *lbs. by the steel-yard; Weight of* 100 *feet,* 3 *lbs.* $3\frac{1}{2}$ *oz. by the scales. Mean vertical Strength, from* 9 *Experiments,* 630 *lbs.*

Weight at A.	*Weight at B.*	*Weight at C.*	*Weight at D.*	*Distance of C from the Ground.*		*REMARKS.*
	lbs.	lbs.	lbs.	ft.	in.	
Fixed	0	0	0	15	6	On account of the length of the wire the curvature was measured from the ground; which latter was about 22 feet from the horizontal line, between the props or points of suspension.
do.	28	14	28	4	$0\frac{1}{2}$	
do.	28	17	28	3	4	
do.	28	19	28	3	0	
do.	28	20	28	2	10	
do.	28	21	28	2	$5\frac{1}{2}$	
do.	28	22	28	2	4	Removed the weights and re-tightened the wire.
do.	0	0	0	16	8	
do.	28	0	28	9	1	
do.	28	14	28	4	8	
do.	28	17	28	—	—	Broke the wire; not at a joint.

This Experiment was made at Ellesmere; the points of suspension were at one end a building, at the other a tree.

3. The nine Experiments from which the mean vertical strength of 630 lbs. was deduced, are as follow:—

	lbs.
1st broke with	616
2d	616
3d	620
4th	652
5th	616
6th	637
7th	616
8th	646
9th	651
	9) 5670
Mean of 9 Experiments	630 lbs.

The wire broke in these Experiments at joints or unsound places.

The mean of twelve other Experiments, on wires of the same diameter, but of different specimens, was 634 lbs.

EXPERIMENTS

4. *On the Momentum which Wires stretched, as in the preceding Experiments, will bear before Breaking.*

Experiment 1. A piece of wire, which bore vertically 277 lbs., was stretched between two props, 140 feet distant from each other, till the versed sine, or deflection in the centre, was only $4\frac{3}{4}$ inches.

A 5 lb. weight was then tied to a cord, and the other end fastened to the middle of the wire; the length of the cord between the weight and the wire was 10 feet 6 inches. The weight being now lifted up to the level of the wire, it was let fall and struck the ground, but without injuring the wire.

Shortened the cord to 7 feet 7 inches, and proceeded as above: it did not strike the ground, nor did it injure the wire.

With the same length of cord, and a 10 lb. weight instead

of the 5 lb., proceeded in the same manner: struck the ground, but did not break the wire.

But the same weight hung by a string 6 feet 7 inches, let fall as above, broke the wire at a joint.

Note.—The distance of the middle of the wire from the ground was 13 feet 6 inches.

By the laws of falling bodies, we have for the

1st	momentum	$(8 \times \sqrt{10{\cdot}5}) \times {\cdot}5 = 129$
2d	....	$(8 \times \sqrt{7{\cdot}58}) \times {\cdot}5 = 110$
3d	....	$(8 \times \sqrt{7{\cdot}58}) \times 10 = 220$
4th	....	$(8 \times \sqrt{6{\cdot}58}) \times 10 = 204$

As the last momentum is less than the preceding, we may infer that the wire was damaged in the third trial.

Experiment 2. Distance of the props, 31 feet 6 inches. Diameter of the wire, $\frac{1}{10}$th inch. Stretched to within $\frac{1}{8}$th of an inch of a straight line.

A 10 lb. weight was tied to the middle of the wire by a cord 7 feet 9 inches long: it was lifted up to the level of the wire, as in the last Experiment, and then let fall; but it did not break the wire.

A 15 lb. weight was tied and let fall in the same manner, without breaking the wire.

A 20 lb. weight was then tried. It did not break the wire.

A 25 lb. weight being let fall from the same height, *broke the wire*.

Here our four momenta are,

1st	momentum	$(8 \times \sqrt{7{\cdot}75}) \times 10 = 222{\cdot}6$
2d	....	$(1 \times \sqrt{7{\cdot}75}) \times 15 = 333{\cdot}9$
3d	....	$(8 \times \sqrt{7{\cdot}75}) \times 20 = 445{\cdot}2$
4th	....	$(8 \times \sqrt{7{\cdot}75}) \times 25 = 556{\cdot}5$

Comparing these momenta with the direct vertical strength, we have

1st vertical strength......	277 lbs.	momentum	220
2d ditto for wire of $\frac{1}{10}$ inch,	630 lbs.	ditto	556·5

that is, in the 1st Experiment, the number expressing the momentum is less by $\frac{1}{5}$th than the vertical strength; and in the 2d by $\frac{1}{8}$th: but it is probable that in the latter the wire would have been broken with a less weight than 25 lbs.

EXPERIMENTS

5. *Upon the direct Strength of Cohesion of malleable Iron, made at* Messrs. BRUNTON and Co.'s *Patent Chain Cable Manufactory, with an Hydrostatic Machine, or Bramah Press, constructed by* Mr. FULLER. By THOMAS TELFORD, Esq.

EXPERIMENT, No. 1.

Cylindrical Bar of South Wales Iron, manufactured by S. HOMFREY, Esq.

April 5th, 1814.	Length of bar when put in ..	2 feet	$2\frac{3}{4}$ inches.
	Ditto when taken out	2	$6\frac{7}{8}$
	Diameter when put in	0	$1\frac{3}{8}$
	Ditto when taken out	0	$1\frac{1}{8}$

Torn asunder by 43 ton 11 cwt.

EXPERIMENT, No. 2.

Cylindrical Bar of South Wales Iron, manufactured by S. HOMFREY, Esq.

April 5th, 1814.	Length of bar when put in ..	2 feet	$3\frac{3}{8}$ inches.
	Ditto when taken out	2	$6\frac{5}{8}$
	Diameter when put in	0	$1\frac{1}{2}$
	Ditto when taken out	0	$1\frac{1}{4}$

Torn asunder by 52 tons 15 cwt. 1 qr. 10 lbs.
Time, 34 minutes.

EXPERIMENT, No. 3.

Square Bar of Staffordshire Iron.

May 17th, 1814.	Length of bar when put in ..	1 foot	$5\frac{1}{8}$ inches.
	Ditto when taken out	1	$11\frac{1}{4}$
	Side of square when put in ..	0	$0\frac{3}{4}$
	Ditto when taken out	0	$0\frac{6}{10}$

Began to stretch with 12 tons; broke with 15 tons 5 cwt. 3 qrs. 4 lbs. Time, $9\frac{1}{4}$ minutes.

EXPERIMENT, No. 4.

Square Bar of Staffordshire Iron.

May 17th, 1814.	Length of bar when put in ..	1 foot	$7\frac{1}{4}$ inches.
	Ditto when taken out	1	$9\frac{1}{4}$
	Side of square when put in..	0	$1\frac{1}{12}$
	Ditto when taken out	0	$\frac{5}{6}$

Began stretching with 32 tons; broke with 32 tons 6 cwt. 4 lbs. Time, 16 minutes.

EXPERIMENT, No. 5.

Square Bar of Welsh Iron, 1 *inch square.*

May 5th, 1817.	With 18 tons stretched	$0\frac{1}{4}$ inches.
	Ditto 21 tons ditto	$0\frac{1}{2}$
	Ditto 23 tons ditto	$0\frac{3}{4}$
	Ditto 25 tons ditto	1
	Ditto 27 tons ditto	$2\frac{1}{4}$
	Ditto 29 tons ditto	$2\frac{3}{8}$ Broke with this weight.

EXPERIMENT, No. 6.

Bar of Swedish Iron, 1 *inch square.*

May 5th, 1817.	Began to stretch with	17 tons.
	Stretched with......	20 tons $\frac{1}{10}$th inch.
	Ditto with	27 tons $\frac{3}{8}$ths.
	Ditto with	29 tons. Broke at a flaw.

Note.—The above, and following stretchings, were measured on 12 inches in the middle of the bar.

EXPERIMENT, No. 7.

Bar of faggotted Iron, from scrap Iron. By M. Howard, *of Rotherhithe.* 1 *inch square.*

May 5th, 1817.	Began to stretch with	16 tons.
	Stretched with......	20 tons $\frac{3}{8}$ inch.
	Ditto with	25 tons $\frac{3}{4}$
	Ditto with	28 tons $2\frac{3}{8}$
	Ditto with	29 tons. Broke with this weight.

Note.—A similar bar began to stretch with 18 tons, and broke with the same weight as above; viz. 29 tons.

EXPERIMENT, No. 8.

Bar of common Staffordshire Iron, 1 inch square.

May 5th, 1817.	Began to stretch with 19 tons.		
	Stretched with......	24 tons	½ inch.
	Ditto with	28 tons	⅝
	Ditto with	29 tons	⅝
	Ditto with	30 tons	1
	Ditto with	31 tons.	Broke with this weight.

EXPERIMENT, No. 9.

Cylindrical Bar of common Iron, 2 inches diameter.

May 21st, 1817.	With 45 tons	Began to stretch; about $\frac{1}{10}$th of an inch on 12 inches, in the middle. The machine being relieved, the bar shortened $\frac{1}{40}$th of an inch.
	With 50 do.	Stretched ·125 inch; relieved and shortened as before.
	With 55 do.	Do. ·25; do. do.
	With 60 do.	Do. ·26
	With 70 do.	Do. ·375 inch; recovered very little when the machine was relieved.
	With 75 do.	Do. ·544; do. do.
	With $80\frac{1}{10}$ do.	Do. ·75; reduced in diameter to $1\frac{1}{16}$th inch.
	With 85 do.	Do. ·86; no perceptible change.
	With 90 do.	Do. 1·00; do. do.
	With 95 do.	Do. 1·35; reduced in diameter to $1\frac{3}{8}$th inch.
	With 100 do.	Do. 2·2; do. do. to 1½ nearly.

With the last weight the bar gave evident signs of fracture; and, in a few minutes, gradually gave way.

Note.—The whole length of the above bar was 2 feet; and it stretched in

its whole length 2¾ inches; of which 2¼ inches were in 12 inches in the middle part. The whole time of making this Experiment was 3 hours; and it was performed with the utmost care.

The machine was frequently relieved; and, when re-applied, constantly brought up the weight to what it was before, but never exceeded it; which is evidence of its accuracy.

Note.—It is a curious fact, and deserving the attention of philosophers, that frequently, at the moment of rupture, the bar acquires such a degree of heat in the fractured part, as scarcely to enable a person to hold it grasped in his hand without a painful sensation of burning.

6. *Reduction of the above to* 1 *inch square.*

	tons.	cwt.	
No. 1, reduced to 1 inch square, gives	29	6	Welsh.
No. 2, ...	29	16	ditto.
No. 3, ...	27	3	Staffordshire.
No. 4, ...	27	10	ditto.
No. 5, ...	29	0	Welsh.
No. 6, ...	29	0	Swedish.
No. 7, ...	29	0	Faggotted.
No. 8, ...	31	0	Staffordshire.
No. 9, ...	31	16	
9)	263	11	
Mean strength of an inch square bar	29	$5\frac{2}{3}$	

Reducing the experiments reported at page 259 on wire to the same standard, we find the strength equal to $35\frac{9}{11}$ tons to the square inch.

By comparing this mean result with that deduced from the following Experiments by Captain Brown, a considerable difference will be found, which it is important to explain, and which appears to me to be attributable to the operation of the two machines; viz. the one overrating its power, and the other falling short of it. Messrs. Brunton and Co.'s machine is an hydraulic press; in which, with high pressures, there is necessarily great friction between the piston leather and the barrel, and the power of the machine is opposed both to the resistance and friction: therefore, if we estimate the whole as opposed to resistance only, the strain will be over-rated. In Captain Brown's machine the case is exactly reversed; the friction and inertia having both a tendency to make its apparent power too small.

EXPERIMENTS

7. *On Iron Bars and Cables, made at the Patent Iron Cable Manufactory of* Captain S. Brown, *Mill Wall, Poplar, on a Machine which acts on the Principle of the Weigh Bridges. From a Report presented to the Author by the above Gentleman.*

(COPY.)

Mill Wall, Poplar, 28th May, 1817.

Experiments on different Descriptions of Iron.

EXPERIMENT, No. 1.

A bar of Swedish iron, 3 feet 6 inches long, $1\frac{5}{16}$ inch square, required a strain of 40 tons 19 cwt. to tear it asunder in a straight line. It stretched, during the operation, $\frac{3}{16}$ths of an inch. No perceptible alteration in the general appearance of the bar, except at the place of rupture, where it was reduced to $1\frac{1}{16}$th of an inch.

The particles remarkably small and close, of a whitish grey colour; not the least heated in the operation.

EXPERIMENT, No. 2.

Another piece, 3 feet 6 inches long, same bar, required a strain of 39 tons 15 cwt. to tear it asunder in a straight line. It stretched $\frac{1}{4}$th of an inch, the bar being torn into cracks in various places. It reduced to $1\frac{1}{16}$th of an inch at the place of rupture. The particles remarkably close and small, as before, intermixed with a few fibrous specks.

Colour, whitish grey; not heated at the time of rupture.

EXPERIMENT, No. 3.

A Swedish bar, 3 feet 6 inches long (different mark), $1\frac{3}{16}$th inch square, required 33 tons 10 cwt. to tear it asunder in a straight line. This bar was exceedingly soft and ductile,

having stretched 3 inches in the operation; and reduced at the place of rupture to $\frac{7}{8}$ths of an inch. It broke extremely fibrous, exhibiting no particles. The complexion silvery: very much heated at the place of rupture.

EXPERIMENT, No. 4.

A bolt of Russia old sable, marked C C N, 3 feet 6 inches long, $1\frac{5}{16}$th inches diameter, required a strain of 36 tons 2 cwt. to tear it asunder in a straight line. This iron, very soft and ductile, stretched $2\frac{1}{4}$ inches, and reduced at the place of rupture to 1 inch in diameter. This iron appeared at the place of rupture in the form of a scarf, as if it had been cut with a pair of shears; the surface so smooth, that there was no appearance of fibres or particles: its fibrous quality was, however, sufficiently indicated by the whole appearance of the bolt.

EXPERIMENT, No. 5.

A bar of Welsh Iron, denominated No. 3; 3 feet 6 inches long, $1\frac{1}{4}$ inch square, required a strain of 38 tons 1 cwt. to tear it asunder. This iron possessed considerable ductility, but reduced in diameter more gradually than in the two preceding Experiments. It stretched 2 inches; and was reduced at the place of rupture to $1\frac{1}{16}$th inch. The complexion of this iron, when looking directly down upon the place of rupture, was a dingy blue, and when held horizontally to the light and viewed obliquely, bright and fibrous, though not so white or silvery as the foreign iron. Very much heated at the place of rupture.

EXPERIMENT, No. 6.

A bar of common Welsh Iron, 3 feet 6 inches long, $1\frac{1}{8}$ inch square. It required a strain of 31 tons. This bar had little ductility, and suffered no general derangement in the operation. It broke directly across the bar; and measured, at the place of rupture, $1\frac{1}{16}$th inch. The particles of this iron were fine, and exceedingly condensed, resembling steel;

and there appeared nothing of a fibrous nature in it: indeed, its complexion and texture seemed to be at variance with the general rules for judging of the quality of iron. Its measure of strength, however, was most accurately ascertained.

EXPERIMENT, No. 7.

A highly interesting one. A bolt of Welsh iron denominated No. 3, 12 feet 6 inches long, 2 inches in diameter, required a strain of 82 tons 15 cwt. to tear it asunder. When subject to a strain of 68 tons, it stretched 3 inches, and was reduced to $1\frac{15}{16}$ths inch in diameter. When the strain was increased to 74 tons 15 cwt., it had stretched 6 inches, and was reduced $\frac{1}{8}$th of an inch gradually in the diameter. With 82 tons it stretched 14 inches. With 82 tons 15 cwt. the bolt broke about 5 feet from the end, the levers being exactly balanced. It had stretched during the whole process $18\frac{1}{2}$ inches; and measured at the place of rupture $1\frac{5}{8}$ inch in diameter.

EXPERIMENT, No. 8.

A bar of blistered steel, made at Sheffield, $1\frac{1}{4}$ inch square, 3 feet 6 inches long, required a strain of 23 tons to tear it asunder in a straight line. It suffered no general derangement in the process, but broke directly across the bar. The particles large, angular, and bright. Reduced nothing at the place of rupture. Not the least heated in the process.

EXPERIMENT, No. 9.

A bar of cast steel, $1\frac{5}{16}$ths inch square, 3 feet 6 inches long, required a strain of 48 tons 10 cwt. to tear it asunder. This bar suffered no visible derangement; neither reduced nor lengthened in the process. The rupture exactly transverse: the texture close, and of a dingy grey complexion.

EXPERIMENT, No. 10.

A bar of cast iron, Welsh pig, 1¼ inch square, 3 feet 6 inches long, required a strain of 11 tons 7 cwt. to tear it asunder: broke exactly transverse, without being reduced in any part; quite cold when broken; particles fine; dark bluish grey colour.*

SAM. BROWN.

EXPERIMENT, No. 11.

A bolt of Welsh iron, 1⅝ inch diameter, 5 feet in length, was torn asunder by a force of 43½ tons.

With 28 tons its diameter was reduced to 1·4 inches.

With 35 tons - - - - - - 1·35 inches.

With 40 tons - - - - - - 1·30 inches.

With 43 tons the bolt broke, having lengthened during the Experiment 7 inches. Considerable heat about the section of fracture.

This is the only one of the above Experiments at which I was present.

8. *Reducing the above to inch square.*

			tons.
No. 1	Swedish iron	1 square inch,	23·77
No. 2	Ditto	ditto	23·19
No. 3	Ditto	ditto	23·75
No. 4	Russia	ditto	26·55
No. 5	Welsh	ditto	24·35
No. 6	Ditto	ditto	24·90
No. 7	Ditto	ditto	26·33
No. 8	Blistered steel	ditto	14·27†
No. 9	Cast steel	ditto	27·92†
No. 10	Cast iron, Welsh pig	ditto	7·26
No. 11	Welsh	ditto	26·34

Mean of the first 7 and the 11th .... 25 tons.

The mean of Mr. Telford's Experiments is . 29½ tons.

* The machine on which these Experiments were made is the property of Captain S. Brown, the inventor of the iron cables; and is constructed on the same principle as the weigh-bridges.

† It is obvious, by referring to the Experiments by Mr. Rennie, (Art. 10,) that these two specimens of steel must have been defective.

And the mean of the two, 27 tons, nearly, which may be safely assumed as the medium strength of an iron bar 1 inch square.*

Additional Experiments on the Direct Cohesive Powers of Cast Iron. By Captain S. Brown.

			tons.	cwt.
$1\frac{1}{4}$ inch square bar, broke with			11	5
$1\frac{1}{4}$	do.	do.	14	5
$1\frac{3}{8}$	do.	do.	14	5
$1\frac{1}{4}$	do.	do.	16	0

These reduced to square inch bars give,

			tons.
1.	Cohesive	strength	7·20
2.	do.	do.	9·12
3.	do.	do.	7·53
4.	do.	do.	10·24
No. 10 preceding....			7·26
			5)41·35

Mean...... 8·27 tons, or 18564 lbs.

* The great difficulty of producing such immense strains is extremely likely to give rise to some inaccuracy in estimating the power; and I think it highly probable, as already stated, that while Captain Brown's machine may shew less than its real force, that of Messrs. Brunton may be overrated. They are both very ingenious: but in the latter, the ratio between the power and weight is extremely great; every pound in the scale increasing the power more than 16,000 times; and in the latter, the inertia is immense; and therefore difficult to estimate with accuracy.

9. For the following Experiments on the Strength of Yorkshire Iron, the Author is indebted to his friend M. J. Brunel, Esq. They were made on bars reduced in the centre part (per hammer) to $\frac{3}{8}$ths and $\frac{4}{8}$ths, or $\frac{1}{2}$ inch square; but the results are all reduced to rods of 1 inch square.

Experiments of the Direct Cohesive Power of Hammered Iron.
By M. J. Brunel, Esq.

Iron denoted *best* $\frac{3}{8}$ths in the middle.			Iron denoted *best best* $\frac{3}{8}$ths in the middle.			Iron denoted *best* $\frac{1}{2}$ in the middle.		
No.	*Began to Stretch.*	*Breaking Weight.*	No.	*Began to Stretch.*	*Breaking Weight.*	No.	*Began to Stretch.*	*Breaking Weight.*
	Tons per inch.	Tons per inch.		Tons per inch.	Tons per inch.			Tons.
1	21	29·8	1	28·16	35·12	1		27
2	24	32	2	27·4	36·4	2		31·12
3	18·15*	25*	3	24·16	32·16	3		31·62
4	22	34·19	4	27·16	33·10	4		32·25
5	20	34·6	5	22·15	31·14	5		32·75
6	20	28·2	6	25·18	31·15	6		30·00
7	23·2	28·2	7	22·3	31·9			
8	24	31·6	8	21·9	29·6			
9	26·9	32·11	9	23·9	31·7			
10	23·1	28·12	10	21·9	30·7			
Mean	22·2	30·4		24·4	32·3			30·8

The mean strength of these bars considerably exceeds that drawn from the preceding Articles; a circumstance which may, it is presumed, be explained from the circumstance of their having been reduced per hammer.

* The Experiment No. 3 of the first series was obviously defective.

EXPERIMENTS

10. *On the Strength of Direct Cohesion of various Metals.* By G. Rennie, Esq.

			Reduced to in. sq. lbs.	tons.
1	¼ inch	cast iron bar, horizon cast, 1166	1193	8·51
2	do.	vertical cast, 1218		
3	do.	cast steel previously tilted	8391	59·93
4	do.	blistered steel, reduced per hammer	8322	59·43
5	do.	shear ditto ditto	7977	56·97
6	do.	Swedish iron ditto	4504	32·15
7	do.	English ditto ditto	3492*	24·93
8	do.	Hard gun metal.................	2273	16·23
9	do.	Wrought copper reduced per hammer	2112	15·08
10	do.	Cast copper	1192	8·51
11	do.	Fine yellow brass	1123	8·01
12	do.	Cast tin	296	2·11
13	do.	Cast lead	114	0·81

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11. *Experiments on the Strength of Chain, made of varions Descriptions of remanufactured foreign and English Iron, performed 2d September,* 1816, *at* Captain Brown's *Manufactory.*

		tons.	cwt.
1½ inch........	Old sable, 1½ inch square bars, cut into pieces 2 feet long, piled and rolled into bolts of 1½ inch	73	10
1½ inch........	Old sable, ditto, ditto.................	80	0
1½ inch..	Gurcoft new sable, ditto, ditto	71	0
1½ inch........	Keiolsken, Archangel, inch square bars, cut into lengths of two feet, piled, and rolled into bolts	71	0

* This specimen of English iron, it appears, must have been defective; its resistance being much inferior to the general mean of those above reported, notwithstanding it was reduced per hammer, which ought to have given it additional strength.

		tons.	cwt.
1½ inch........	Old bolts, found promiscuously, piled and faggotted by hand-hammers at my works	71	10
1½ inch full	English bars, piled and rolled..........	86	0
1½ inch bare....	Ditto, ditto	80	0

12. *Further Experiments, made* 13*th September*, 1816.

	tons.	cwt.
1½ inch Old Dutch bolts, faggotted by hand-hammers at my works	71	0
1½ inch No. 1, ⅝ square (Welsh iron,) hammered into blooms, and rolled into bolts, at the King and Queen works	78	10
1½ inch No. 2, ¾ inch square (Welsh), manufactured as above	73	5
1½ inch No. 4, Welsh iron, faggotted by hand-hammers at my works	88	10
1½ inch No. 6, ⅝in. square ditto, rolled, but not hammered, at the King and Queen works	76	0
1½ inch King and Queen scrap iron	80	5

The links of these chains were of an oval-like form, 6 inches in the clear.

S. BROWN.

13. The mean of these experiments gives 76 tons for the strength of a double bolt of 1½ inch diameter, *in the cable form*, which corresponds to about 21½ tons per square inch. Now, by the same machine we have found the mean strength of wrought iron, per square inch, to be 25 tons; therefore, the strength of iron in the cable form is to that of the simple bolt in about the ratio of 43 to 50. But in these cables the links were without stays: when these are introduced, as in Brunton's patent cable, the strength is very nearly equal to that of the iron in the simple bar form; so that the stay may be said to increase the strength by about one-sixth part; at the same time, however, it must be considered that the weight is also increased, although perhaps in a less ratio.

14. *Table shewing the different kinds of best Bower Cables at present employed in the British Navy, with the corresponding Iron Cables.*

Rates of Ships.	*Best bower hempen cables, 100 fathoms.* Sizes, circum.	Weight.			*Number of threads in each.*	*Breaking strain by experiment.*			*Diameter and weight of the bolt of the iron cable substituted for the preceding.*
	inches.	cwt.	qr.	lb.		tons.	cwt.	qr.	
First rate, large ...	25	114	2	7	3240	—	—	—	
middle	24	105	2	17	2988	—	—	—	
small ...	23	96	2	27	2736				2⅛ inches.
Second rate.........	23	96	2	27	2736	114	0	0	218 cwt.
Third, large	23	96	2	27	2736				
small	22	89	0	12	2520	89	0	0	2 inches.
Fourth, 60 guns ...	21	80	0	22	2268				186 cwt. 2 qr.
58 ditto ...	19	66	0	21	1872	—	—	—	1⅞ inch.
50 ditto ...	18½	62	1	14	1764	—	—	—	170 cwt. 2 qr.
Fifth, 48 ditto ...	18	58	2	6	1656	63	0	0	
46 ditto, 42 ditto	17½	56	0	1	1584	—	—	—	1¾ inch. 145 cwt. 3 qr.
Sixth, 28 guns ...	14½	38	0	21	1080	40	0	0	1⅜ inch. 87 cwt. 2 qr.
Ship sloop	13½	33	0	10	936	—	—	—	1¼ inch.
Brig, large	13½	33	0	10	936	—	—	—	74 cwt. 3 qr.
Ditto, small.........	11	21	2	15	612	—	—	—	1⅛ inch. 61 cwt. 1 qr.

From the above table the immense advantage of iron cables will be distinctly seen, and particularly when we consider that a hempen cable, on a rocky bottom, is destroyed in a few months, while the other will sustain no perceptible injury.

The following are the results of a set of experiments on ropes of different thicknesses, made and furnished by John Knowles, esq.:

in.		*ton.*	*cwt.*	*qr.*	*lbs.*
4	circumference	4	19	0	0.
3½	ditto	3	12	1	0
3	ditto	2	10	1	0

15. *Experiments on the Transverse Strength of Cast Iron.* By Mr. Banks. (See his Treatise on the Power of Machines.)

No.	*Length.*		*Depth.*	*Breadth.*	*Breaking Weight.*	
1	3 ft.	0 in	1 in	1 in	756 lbs.	mean 756.
2	3	0	1	1	756	
3	2	6	1	1	1008	
4	3	0	1	1	735½	average of 3 experiments.
5	3	0	1	1	963	mean 972.
6	3	0	1	1	958	
7	3	0	1	1	994	
8	3	0	1	1	864	mean 869.
9	3	0	1	1	874	

In the above experiments, the ends of the bars were supported on two props, and the weight applied in the centre.

The following experiments are similar to the above, but more varied:

EXPERIMENTS

16. *On the Transverse Strength of Cast Iron Bars.* By G. Rennie, Esq.

No.	Description of Bar.	Weight of Bar. lbs.	oz.	Distance of Bearings ft.	in.	Breaking Weight. lbs.
1	Bar of 1 inch square	10	6	3	0	897
2	Ditto of 1 inch ditto	9	8	2	8	1086
3	half the above bar			1	4	2320
4	Bar of 1 inch square through the diagonal	2	8	2	8	851
5	Half the above bar			1	4	1587
6	Bar of 2 in. deep, by ½ inch thick	9	5	2	8	2185
7	Half the above bar			1	4	4508
8	Bar 3 inches deep, by ⅓ inch thick	9	15	2	8	3588
9	Half the above bar			1	4	6854
10	Bar 4 inches, by ¼ inch thick......	9	7	2	8	3979
	Equilateral triangles, with the angle up and down					
11	Edge or angle up..............	9	11	2	8	1437
12	angle down............	9	7	2	8	840
13	Half the first bar..............			1	4	3059
14	Half the second bar............			1	4	1656
	A feather edged, or ⊥ bar, was cast, whose dimensions were					
15	2 inches deep, by 2 wide, edge up...................	10	0	2	8	3105
16	Half of ditto..................					

N.B. All the above bars contained the same area, though differently distributed as to their forms.

17. *Experiments made on Bars of 4 inches deep by ¼ inch thick, by giving it different forms, the bearing at 2 ft. 8 in. as before.*

		lbs.
17	Bar formed into a semiclipse weighed .... 7lbs.	4000
18	Do. parabolic on its lower edge	3860
19	Do. of 4 inches deep, by ¼ inch thick	3979

18. *Experiments on the Transverse Strain of Bars, one end made fast, the weight being suspended at the other at 2 feet 8 inches from the bearing.*

		lbs.
20	An inch square bar bore	280
21	A bar 2 inches deep, by ½ inch thick	539
22	An inch bar, the ends made fast	1173

23 The paradoxical conclusion of Emerson was tried, which states, by cutting off a portion of an equilateral triangle (see page 114 of Emerson's Mechanics), the bar is stronger than before, that is, a part stronger than the whole. The ends were loose, at 2 feet 8 inches apart, as before. The edge from which the part was intercepted was lowermost; the weight was applied on the base above; it broke with 1129 lbs., whereas in the other case it bore only 840

19. We have given the above experiment as it is reported by Mr. Rennie; but it is at variance, as well as experiment 11, 12, 13, 14, with all the similar experiments on wood, reported in pages 60, 61, and 103.*

20. If we take Mr. Banks's experiments, and select from Mr. Rennie's those which most resemble them, and thence compute the constant value of S, as in the table of data on woods, page 196, we shall have

* Mr. Tredgold has lately made some experiments on triangular bars of cast iron, and finds that while the elasticity remains perfect, the same weight produces the same flexure, whether the angle or the base be upwards. The breaking weights, in the two cases, did not differ more than one-tenth, when the fracture was at the middle of the length in both trials, and the greater weight was required when the angle was upwards. The bars were supported at the ends. Mr. Duleau had previously ascertained that a triangular bar of wrought iron presents the same resistance, whether it be supported on one of its faces, or one of its arrises.—*Essai Théorique et Expérimental sur la Résistance du Fer Forgé*, pp. 26, 27.

Banks's Exper. 1 and 2,	$S = \frac{l\,W}{4\,a\,d^2}$	$= 6804.$
Exper. 3	$S = \frac{l\,W}{4\,a\,d^2}$	$= 7560.$
Exper. 4	$S = \frac{l\,W}{4\,a\,d^2}$	$= 6619.$
Mean of 5, 6, 7	$S = \frac{l\,W}{4\,a\,d^2}$	$= 8748.$
do. of 8 and 9	$S = \frac{l\,W}{4\,a\,d^2}$	$= 7821.$
Mean value of S		$= 7510.$
Rennie's Exper. 1	$S = \frac{l\,W}{4\,a\,d^2}$	$= 8073.$
Exper. 2	$S = \frac{l\,W}{4\,a\,\delta^2}$	$= 8688.$
Exper. 3	$S = \frac{l\,W}{4\,a\,\delta^2}$	$= 9280.$
Mean value of S =		8680.
Mean from the two sets	-	8095.

If we compare this general mean with the value of S for oak 2d specimen, table of data, viz. 1672, we find it to be nearly five times greater, whereas the cohesive power is not double; the value of C for the oak being 10853, and for the cast iron, from Mr. Rennie's experiments, 19062; and from Captain Brown's, after allowing for the underrating the machine, 17612, or 18337, taking the mean of both. Which shews that this material is much more incompressible than oak; and, consequently, that its neutral axis is much higher, and which as we have seen (art. 19.) may account for the singular results obtained by Mr. Rennie, on triangular iron bars.*

* On the application of cast iron for all kind of mechanical and architectural purposes, the reader should consult the excellent treatise on this subject by Mr. Tredgold.

EXPERIMENTS

21. *On the Transverse Strength of Malleable Iron.*

Not having been able to obtain any satisfactory results relative to the transverse strength of malleable iron, I applied to General Millar, who has the superintendence of the Carriage Department in the Royal Arsenal, who, very obligingly, not only gave the requisite permission, but himself superintended the following experiments made on bars an inch square, 3½ feet in length, rested on props 3 feet asunder.

1. Swedish iron 3 feet, bearing 1 inch square, with

560 lbs.	deflected in the	middle	·25 inch.
716 lbs.	do.	do.	·375 inch.
884 lbs.	do.	do.	·500 inch.

This bar being now unloaded, it was found to recover its rectilinear form, its elasticity having sustained no injury.

2. Swedish iron 3 feet, bearing 1 inch square, with

560 lbs.	deflected in the	middle	·30 inch.
884 lbs.	do.	do.	·40 inch.
1008 lbs.	do.	do.	·55 inch.
1120 lbs.	do.	do.	1 inch.

The elasticity was here obviously destroyed.

1232 lbs.	do.	do.	1⅜ inch.
1288 lbs.	do.	do.	2¾ inch.

3. King and Queen, or faggoted iron.

560 lbs.	deflected in the	middle	·25 inch.
716 lbs.	do.	do.	·50 inch.
884 lbs.	do.	do.	1·25 inch.

Elasticity destroyed.

Taking the mean from the two latter, we have 1000 lbs. for the weight with which a bar of wrought iron, 3 feet long and 1 inch square, has its elasticity destroyed.

If, with this number, we employ the formula $\frac{l\,W}{4\,a\,d^2}$, (as in art. 20) we find,

$$S = \frac{l\,W}{4\,a\,d^2} = 9000.$$

Which is about eight times the mean value of the best fir; while the specific gravity is about eleven times that of the same wood, and the cohesive power about $5\frac{4}{5}$th times. If, therefore, with the same length, we employ weight for weight in fir and iron, the dimension of the bar must be greater in the former than in the latter, in the ratio of $\sqrt{11} : 1$; but with different sized similar beams of the same length, the strength varies as $a\,d^2$, or as the cube of the side; that is, as $11^{\frac{3}{2}} : 1$, or as 36 to 1 nearly: but the strength of iron to that of wood, in bars of equal dimensions, is as 8 : 1; therefore the strength of fir to that of iron exposed to a like transverse strain, and having weight for weight, is as 36 to 8, or as 9 to 2, or $4\frac{1}{2}$ to 1 nearly.

Experiments on the Transverse Strength of Steel.
By M. DULEAU.*

Description of Specimens.	*Distance between the Supports.*	*Breadth.*	*Depth.*	*Deflection with a Weight of 10 Kilogrammes.*
	Mètres.	Millimètres.	Millimètres.	Millimètres.
Cast steel, English, marked HUNTSMAN; perfectly regular, untempered, but brittle	0·98	5·9	13·3	8·4
German steel (of cementation), marked FORTSMAN, and three deer heads, used for razors; dimensions irregular..	0·68	7·8	14·5	2·1
Same kind of steel	1·845	25·7	21·6	2·8
Same piece, on edge ..	1·845	21·6	25·7	2·2
Same kind of steel	1·845	21·9	28·5	1·8
Do. do.	1·35	54·8	25·5	0·55
Same piece, on edge ..	1·35	25·5	54·8	0·27
Same kind of steel	1·35	26·6	52·0	0·3

22. *Experiments on the Resistance of ½ inch Iron Bars to a wrenching force.*

The following experiments were made by George Rennie, esq., and were published by him in the Phil. Trans. Part I., for 1818. The apparatus consisted of a wrought iron lever, 2 feet long, having an arched head of about 60°, and four feet diameter, of which the lever represented the radius: the centre round which it moved had a square hole, made to receive the end of the bar to be twisted. The lever was balanced, and a scale hung on the arched head; the other end of the bar being fixed in a square hole, in a piece of iron, and that again in a vice. The undermentioned weights represent the quantity of weight put into the scale.

* *Essai sur la Résistance du Fer Forgé*, p. 38 et 39. The rest of Duleau's experiments on steel are described in Tredgold's Essay on Cast Iron, art. 68[1], second edition.

EXPERIMENTS

23. *On twists close to the bearing, cast horizontal.*

No.		lbs.	oz.	
1	¼ inch bars, twisted as under, with	10	14	in the scale
2	¼ ditto, bad casting............	8	4	
3	¼ ditto	10	11	
	Average	9	15	

Cast vertical.

4	¼	10	8
5	¼	10	13
6	¼	10	11
	Average	10	10

24. *On twists of different lengths.*

Horizontal cast.

No.		Weight in scale. lbs.	oz.
7	¼ by ½ long	7	3
8	¼ by ¾ ditto..................	8	1
9	¼ by 1 inch ditto..............	8	8

Vertical.

10	¼ by ½ ditto..................	10	1
11	¼ by ¾ ditto..................	8	9
12	¼ by 1 inch ditto..............	8	5

Cast horizontal, twists at 6 inches from the bearing.

13	¼ by 6 inches long	10	9
14	¼ by ditto ditto	9	4
15	¼ by ditto ditto	9	7

25. *Twists of ½ inch square bars, cast horizontally.*

		qrs.	lbs.	oz.	
16	½ close to the bearing	3	9	12	end of the bar hard.
17	½ ditto............	2	18	0	middle of the bar.
18	½ at 10 inches from bearing, lever in the middle	1	24	0	

26. *On twists of different materials.*

These experiments were made close to the bearing, and the weights were accumulated in the scale until the substances were wrenched asunder:

No.		*lb.*	*oz.*
19	Cast steel	19	9
20	Sheer steel	17	1
21	Blister steel	16	11
22	English iron	10	2
23	Swedish iron	9	8
24	Hard gun metal	5	0
25	Fine yellow brass	4	11
26	Copper	4	5
27	Tin	1	7
28	Lead	1	0

It will of course be understood that these experiments give only the relative resistance to torsion, and not the actual resistance. On this subject the reader should consult *Tredgold*'s "Practical Essay on the Strength of Cast Iron;" art. 222.

27. *Experiments by* GEORGE RENNIE, Jun., Esq., *on Resistance of Cast Iron to Pressure, from Phil. Trans. for* 1818.

Size of the prism. Side of base.	Height.	*Specific Gravity.*	*Crushing Weight.*	*Mean from each set.*	*REMARKS.*
inch.	inch.		lbs.	lbs.	
$\frac{1}{8}$	$\frac{1}{8}$	7033	1454	1440	These specimens were from one block.
Do.	Do.	Do.	1416		
Do.	Do.	Do.	1449		
$\frac{1}{8}$	$\frac{2}{8}$	6977	1922	2116	Iron from a block.
Do.	Do.	Do.	2310		
Do.	$\frac{3}{8}$	Do.	2363	1758	These specimens were from the same block.
Do.	$\frac{4}{8}$	Do.	2005		
Do.	$\frac{5}{8}$	Do.	1407		
Do.	$\frac{6}{8}$	Do.	1743		
Do.	$\frac{7}{8}$	Do.	1594		
Do.	$\frac{8}{8}$	Do.	1439		
$\frac{1}{4}$	$\frac{1}{4}$	6977	10561	9773	These specimens were from the same block as above.
Do.	Do.	Do.	9596		
Do.	Do.	Do.	9917		
Do.	Do.	Do.	9020		
$\frac{1}{4}$	$\frac{1}{4}$	7113	10432	10114	These specimens were from horizontal castings.
Do.	Do.	Do.	10720		
Do.	Do.	Do.	10605		
Do.	Do.	Do.	8699		
$\frac{1}{4}$	$\frac{1}{4}$	7074	12665	11136	These specimens were vertical castings.
Do.	Do.	Do.	10950		
Do.	Do.	Do.	11088		
Do.	Do.	Do.	9844		
Do.	Do.	Do.	11006		
$\frac{1}{4}$	$\frac{1}{2}$	7113	9455	9414	Horizontal casting.
Do.	Do.		9374		
$\frac{1}{4}$	$\frac{1}{2}$	7074	9938	9982	Vertical casting.
Do.	Do.		10027		
$\frac{1}{4}$	$\frac{3}{8}$	7113	9006		Horizontal castings.
Do.	$\frac{5}{8}$	Do.	8845		
Do.	$\frac{6}{8}$	Do.	8362		
Do.	$\frac{7}{8}$	Do.	6430		
Do.	$\frac{8}{8}$	Do.	6321		
$\frac{1}{4}$	$\frac{3}{8}$	7074	9328		Vertical castings.
Do.	$\frac{5}{8}$	Do.	8385		
Do.	$\frac{6}{8}$	Do.	7896		
Do.	$\frac{7}{8}$	Do.	7018		
Do.	$\frac{8}{8}$	Do.	6430		

28. *Similar Experiments on different Metals.*

Size of the prism. Side of base.	Height.	Specific Gravity.	Crushing weight.	Mean from each set.	REMARKS.
inch.	inch.		lbs.		
¼	¼	Cast copper.	7318		Crumbled by the pressure.
Do.	Do.	Brass.	10304		Fine yellow brass reduced $\frac{1}{10}$th with 3213 lbs. ½ with 10304lbs
Do.	Do.	Wrought copper.	6440		Reduced $\frac{1}{16}$ with 3427 lbs. ⅛ with 6440.
Do.	Do.	Cast tin.	966		Reduced $\frac{1}{16}$ with 552; ⅓ with 966.
Do.	Do.	Cast lead.	483		Reduced ½ with 483.

In these experiments, after the metals had been compressed to a certain extent, the resistance is stated to have been enormous.

29. *Experiments on the resisting power of various Materials to a crushing force.*

		Specific Gravity.	Crushing Weight.
			lbs.
1	Elm, cube of one inch		1284
2	American pine do.		1606
3	White deal do.		1928
4	English oak do.		3860
5	Portland stone, 2 inches long		805
6	Statuary marble, 1 inch		3216
7	Craigleith do.		8688

		Specific Gravity.	*Crushing Weight.* lbs.
8	Chalk, cube of 1½ inch		1127
9	Brick, pale red do.	2085	1265
10	Roe stone, Gloucestershire do.		1449
11	Red brick do.	2168	1817
12	Do. Hammersmith paviers' do.		2254
13	Burnt do. do.		3243
14	Fire brick do.		3864
15	Derby grit do.	2316	7070
16	Do. another specimen do.	2428	9776
17	Killaly white freestone do.	2423	10264
18	Portland do.	2428	10284
19	Craigleith white freestone do.	2452	12346
20	Yorkshire paving, with the strata do.	2507	12856
21	Do. do. against strata do.		12856
22	White statuary marble do.	2760	13632
23	Bramley Fall sandstone do.	2506	13632
24	Do. against strata do.		13632
25	Cornish granite do.	2662	14302
26	Dundee sandstone do.	2530	14918
27	Portland, a two-inch cube	2423	14918
28	Craigleith, with the strata, 1½ inch cube	2452	15560
29	Devonshire red marble do.		16712
30	Compact limestone	2584	17354
31	Granite Peterhead		18636
32	Black compact limestone	2598	19924
33	Purbeck	2599	20610
34	Black Brabant marble	2697	20742
35	Freestone, very hard	2528	21254
36	White Italian marble	2726	21783
37	Granite, Aberdeen, blue kind................	2625	24556

30. *Experiments on the Strength of Roman Cement.*

I am indebted for the following results to M. J. Brunel, esq., who made them in reference to the construction of the tunnel at present in progress under the Thames.

1. Against a brick-wall a brick was attached by cement, its broadest surface to the wall, and with its length vertical;

to this brick another was added; to this a third; and so on till 13 bricks were thus cemented to each other: to the 13th brick another was added endways; and, lastly, a 15th brick to the end of this, in the same position as the first 13. The cement supported this length of column without any appearance of breaking. Two bricks were then laid on the farthest extremity; and, lastly, four others in front of these; in laying on the last brick the column or arm broke at the wall.

2. In this experiment 12 bricks were cemented to each other exactly as above; and then 9 bricks more were laid on, viz. by placing one over each of the last seven; and, lastly, two at the farthest extremity. The arm was left in this state without breaking.

These experiments were made with Parker and White's cement, which was perfectly dry in both cases before the additional bricks were added.

3. Eleven bricks were attached in the same manner, and several weeks after, 21 bricks were piled upon its farthest extremity. Adding the last brick caused the arm to break off at the wall.

4. Eleven bricks were attached to the wall edgeways; in this state the arm supported 4 bricks, and then broke at the wall.

These two experiments were made with Messrs. Turner and Montague's cement.

5. A column was built 6 feet high and 14 inches square, and when dry was laid lengthways on two props, 5 feet 6 inches asunder; in this position a weight of 896 lbs. was laid over the centre, which it supported without breaking; being at the time of writing this still bearing this load.

6. Exactly the same experiment was tried on a column, using half cement, and half sand; this bore the same weight for half an hour, and then broke.

These experiments were made with Mr. Shepherd's cement.

It may be proper to add, that in every case of fracture the brick itself gave way before the cement.

31. *Comparison of certain of the preceding Experimental Results with those derived from theoretical Computation; principally in reference to the construction of Suspension Bridges.*

In Experiment No. 2, page 255, it appears that a piece of wire, whose vertical strength was 531 lbs., being stretched on props 31·5 feet apart, and having a weight of 120·25 lbs. hung at its middle point, had that point deflected 1 foot 10¾ inches, and that it afterwards broke with the addition of 10 lbs. Let us endeavour to compute how much this 10 lbs. exceeded what was absolutely necessary to break the wire: or, which is the same, let there be given the distance of the props, the deflection, and the tension of the wire, to find the weight which, suspended from its middle point, will produce the rupture.

Let A, B, *fig.* 10, *plate* III. represent the two fixed points; C E the deflection; A C B the wire: then it is obvious that the point C is kept in equilibrio by three forces; viz. A C, which denotes the tension of A C, C B, or the equal tension of C B; and the unknown weight, W, *plus* half the weight of the wire, *w*. Now, when three forces, acting on a material point, preserve that point in equilibrio, each of the three forces is equal and directly opposed to the resultant of the other two. If, therefore, C B be produced to meet the vertical A D, D E will denote the resultant of the two forces, T, and $(W + w)$, representing by T the tension of A C: therefore, A D C will be the triangle of forces which keeps the point C in equilibrio; of which the side A D will denote the vertical force or weight, $W + w$; and by the nature of the construction $A\,D = 2\,C\,E$; whence then we have as

$$A\,C : A\,D \text{ or } 2\,C\,E :: T : W + w.$$

$$\text{or, } W = \frac{2\,C\,E \times T}{A\,C} - w.$$

Now, $C\,E = 1{\cdot}8958$ feet, or $2\,C\,E = 3{\cdot}7916$.

Also, $A\,C = \sqrt{(A\,E^2 + E\,C^2)} = 15{\cdot}86$.

And by the data of Experiment 1, $w = \cdot 29$ lbs.

Whence $W = \frac{3 \cdot 7916 \times 581}{15 \cdot 86} - \cdot 29 = 126 \cdot 65$ lbs.

This is about 4lbs. less than the weight found by the experiment; but it is to be observed, that we have taken here the deflection due to the weight of 120¼ lbs.; whereas, although the last deflection is not stated, there is no doubt the wire descended lower after the last weight was on; which, as it would increase A D, would increase also the ultimate weight W: and we should probably with this datum find a difference less than the fraction of a lb. between the experimental and theoretical results.

We may arrive at the same conclusion on principles a little different from the above, and somewhat more general; viz. since the weight W is kept in equilibrio by the tensions of A C and C B; and since this weight, W *plus w,* the weight of the wire, is the only vertical force in the system, if we denote the tension of the wires A C and C B by T and T', and the angles E A C, E B C, by a and a', and resolve these two forces each into its component horizontal and vertical force; we must have the two former equal to each other, and the sum of the other two equal to the sum of the vertical weights, $W + w$; that is, we shall have

$$T \cos. a = T' \cos. a'$$

$$T \sin. a + T' \sin. a' = W + w:$$

from which equations the two tensions, T and T', may be determined, whatever may be the ratio of the two parts A C, C B; but in our case, as these are equal, the first equation disappears, and the second becomes

$$2 T \sin. a = W + w, \text{ or}$$

$$T = \frac{W + w}{2 \sin. a}.$$

Or if T be given, and W required,

$$W = 2 T \sin. a - w.$$

In the experiment above referred to, $T = 531$, and $sin.\ a = \cdot 1195593$, and $w = \cdot 29$.

Whence $W = 2 \times 531 \times \cdot 1195593 - \cdot 29 = 126 \cdot 65$ lbs., as before.

We might proceed in a similar manner to compute the tensions of the extreme points, when there are more than one weight, as in our third and subsequent experiments: but it will be, perhaps, more simple to begin here by computing the tensions of the two adjacent sides, C D and D E (*fig.* 11, *plate* III.); which may be effected precisely in the same manner as in the preceding case. For it is a principle in mechanics, that if a system of forces be in equilibrio, no alteration will take place in that state, by supposing any two or more of its points to become fixed: we may, therefore, suppose the points C and E fixed, and compute the tension of C D, or D E, exactly as above; viz. calling the angle D C E $= a'$, and the centre weight W', and the tension t, we shall have

$$t\ sin.\ a = \tfrac{1}{2} W' + \tfrac{1}{4} w,$$

$$t = \frac{W' + \frac{1}{2} w}{2\ sin.\ a'},$$

where w is the whole weight of the wire: then, having the tension t, the weight W, and the angle D C W, compute the value of the resultant of these two forces, which will obviously be the tension of A C; that is, if we denote this tension by T, we shall have

$$T = \sqrt{\{t^2 + W^2 + 2\ sin.\ a'\ t\ W.\}}$$

In experiment 3, we have $W' = 74$, and $w = 2 \cdot 5625$, and $sin.\ a = \cdot 06685$, whence

$$t = \frac{75 \cdot 2812}{\cdot 1337} = 563 \text{ lbs.}$$

$$\text{And } T = \sqrt{\{563^2 + 77^2 + \cdot 1337 \times 563 \times 77\}} = 573.$$

This gives the tension too little: let us therefore compute the same from the 1st deflection; that is, by resolving T into two forces, the one horizontal, and the other vertical; and

equating the latter with half the sum of the weights, *plus* half the weight of the wire; for as the whole system is retained in equilibrio by the two extreme tensions, the vertical component of each ought to be equal to half the entire vertical force, or half the whole weight. This consideration gives us

$$T \sin. a = \tfrac{1}{2}(W + \acute{W} + \acute{\acute{W}} + w),$$

where a denotes the angle C A c. In the 3d experiment, $a = 7° 32'$ and $\sin. a = \cdot 1311$,

$$\text{Whence } T = \frac{230 \cdot 56}{\cdot 2622} = 879.$$

If now we take the mean of our two results, we shall have

$$\frac{879 + 573}{2} = 726 \text{lbs.}$$

Whereas the vertical strength, as determined from experiment, was 736lbs.

The two different results given by the two methods, shew that the system had assumed a form inconsistent with a perfect state of equilibrium, supposing the several lengths, or distances, A c, c d, &c. to be equal: but it is obvious, that besides the probable unequal extensibility of the wire, that the point C, as the wire stretches, will approach towards A, and recede from the perpendicular; for D being exposed to equal actions on each side, will continue in the same vertical: this will obviously have a tendency to increase the angle a, and decrease the angle $\acute{a}$; and, consequently, to increase the value of the tension computed according to the former method, and to diminish the same according to the latter, and therefore approximate them towards that medium result we have obtained above, which differs only 10lbs. from what was found experimentally; viz. about 1lb. out of 73lbs.

In our 4th experiment, $\acute{a} = 2° 51'$ $\sin. \acute{a} = \cdot 04893$, and $\tfrac{1}{2} w = \cdot 39$lb.

$$t = \frac{76 \cdot 39}{\cdot 09786} = 790 \text{lbs.}$$

And, $T = \sqrt{\{790^2 + 71^2 + \cdot 0944 \times 790 \times 71\}} = 797$ lbs.

According to the second principle, viz.

$$T\, sin.\, a = \tfrac{1}{2}\,(W + \acute{W} + \acute{\acute{W}} + w),$$

we have $W + \acute{W} + \acute{\acute{W}} + w = 218{\cdot}79$, and $sin.\, a = \cdot 12648$; whence

$$\frac{218{\cdot}79}{\cdot 25296} = 864 \text{ lbs.}:$$

the mean of which is 831 lbs. instead of 736 lbs., which is in excess by about $\frac{1}{8}$th part.

32. It will be observed, however, that the methods we have hitherto adopted are only approximative; and we have taken this partial view of the problem, because we conceived that it might be more intelligible to many readers, than if we had entered upon it with all the generality that belongs to the doctrine of equilibrium of flexible bodies: but it may not be amiss to give a sketch of this general method, at least as applied to the action of vertical weights upon a perfectly flexible line.

Here we may suppose any number of weights $W, w, \acute{w}$, &c. $\acute{W}$; and a corresponding number of distances $L, l, \acute{l}, \acute{\acute{l}}$, &c. $\acute{L}$, which may be equal or unequal: the tensions of these lines we may denote by

$$T, t, \acute{t}, \acute{\acute{t}}, \text{\&c. } \acute{T},$$

and their several angles, with reference to a horizontal axis $A\,x$, passing through A, by

$$a, \alpha, \acute{\alpha}, \acute{\acute{\alpha}}, \text{\&c. } \acute{a},$$

and their angles with reference to the other axis $A\,y$,

$$b, \beta, \acute{\beta}, \acute{\acute{\beta}}, \text{\&c. } \acute{b}.$$

Also, let n be the co-ordinate of the point B with reference to $A\,y$, and m its co-ordinate as referred to $B\,y$.

Then if we resolve each of the tensions into its corresponding horizontal and vertical components, we shall have from the theory of equilibrium,

$$T\ cos.\ a + t\ cos.\ \alpha + t'\ cos.\ \alpha' + \&c.\ T'\ cos.\ a' = o,$$

$$T\ cos.\ b + t\ cos.\ \beta + t'\ cos.\ \beta' + \&\ T'\ cos.\ b' = o.$$

And by means of the co-ordinates,

$$L\ cos.\ a + l\ cos.\ \alpha + l'\ cos.\ \alpha' + \&c.\ L'\ cos.\ a' = n,$$

$$L\ cos.\ b + l\ cos.\ \beta + l'\ cos.\ \beta' + \&\ L'\ cos.\ b' = m,$$

and by the known property of cosines,

$$cos.^2\ a + cos.^2\ b = 1,$$

$$cos.^2\ a' + cos.^2\ b' = 1.$$

From which six equations the six unknown quantities, viz. T, T', $cos.\ a$, $cos.\ b$, $cos.\ a'$, $cos.\ b'$, may be determined; after having first computed t, t', &c. and $cos.\ \alpha$, $cos.\ \alpha'$, &c. in functions of T, $cos.\ a$, and W, w, w', &c. which may, in all cases, be effected on the general principle of the composition of forces; that is, taking t as the resultant of T and W, t' as the resultant of t and w, and so on.

The computations, however, if the number of weights be considerable, become extremely laborious, and difficult to execute: but if, as in the experiment, we limit the weights to three, and consider the two extreme ones equal to each other, and the points A and B as being situated in the same horizontal line; then, as the several tensions and angles from each extreme are equal, we may reduce the above equations to three; in which, however, we have still to compute $cos.\ \alpha$ in functions of T, $cos.\ a$ and W; on which account we prefer, in this case, retaining the six equations under the form,

$$T\ cos.\ a = t\ cos.\ \alpha,$$
$$T\ cos.\ b = t\ cos.\ \beta + W,$$
$$l\ cos.\ a + l\ cos.\ \alpha = \tfrac{1}{2}\ n,$$
$$T\ cos.\ b = \tfrac{1}{2}\ (W + w + W),$$
$$cos.^2\ a + cos.\ b^2 = 1,$$
$$cos.^2\ \alpha + cos.^2\ \beta = 1.$$

From which these several quantities may be determined, in functions of each other.

If we denote the less deflection c C by d, and the greater D d by $d + d'$, we shall have

$$\frac{d}{l} = cos.\ b, \text{ and } \frac{d}{l} = cos.\ \beta;$$

and substituting these in the first four equations, and denoting the entire weight of the system by π, we shall have, after reduction,

$$\mathrm{T} \surd (l^2 - d^2) = t \surd (l^2 - d'^2)$$
$$\mathrm{T}\, d = t\, d' + \mathrm{W}$$
$$\surd (l^2 - d) + \surd (l^2 - d'^2) = \tfrac{1}{2}\, n$$
$$\mathrm{T}\, d = \tfrac{1}{2}\, l\, \pi.$$

From which we may determine any one of these quantities in terms of the others: but it will be observed here, as in our partial solution, that if we suppose both deflections d and d' as known quantities, we shall have a superfluity of data; viz. we shall have more equations than unknown quantities; and therefore, by assuming values for both these, we may give such as are inconsistent with the other data, and therefore also inconsistent with a state of perfect equilibrium: we ought, therefore, in the solution of these equations, to include one of these quantities with the data, and one with the *quæsiti* of the problem: in which case a rational solution will be obtained.

We shall not attempt the numerical solution of these equations; but the reader who is desirous of doing so will find no other difficulty than what belongs to the algebraical operations: we shall content ourselves with the approximative numbers as above determined, considering it useless to expect a nearer approximation between theory (which is founded on a supposition of a perfect uniformity of matter, and the most accurate mode of action) and experiments, in which every kind of irregularity with regard to the composition of the material, and all the errors of fixing, observing, &c. are presented: indeed the agreement between the two deductions may be considered a confirmation of the correctness of the theory, and of the accuracy with which the

experiments were performed; and on the basis of the two combined every confidence may be placed, as to computation, relative to works which from their magnitude bid defiance to any experiment, except that of their actual construction. It may not, therefore, be amiss to compute from our experimental data, and theoretical principles, the absolute weight which the bridge, to which we have alluded in the beginning of this Appendix, may sustain beyond that of the materials, before any apprehension need be entertained of its giving way.

Computation relative to the Strength of the projected Runcorn Bridge.

32. In this case we must make our computations with reference to the *catenary*, or the curve which a heavy flexible chain will assume when suspended on two fixed points; which, in the present case, we shall suppose to be in one and the same horizontal line.

The properties of this curve are investigated in most treatises of mechanics; we shall not, therefore, retrace steps which have been so often taken, but merely bring under one point of view these several relations; referring such of our readers as may be desirous of actual investigations, to the several works in which they may be found, particularly to Poisson, "Traité de Mécanique," whence the following has been selected:

Let l denote the length of the catenary;

l' the distance of its points of suspension;
c the angle between the tangent at the point of suspension, and the above horizontal line of distance;
A the tension of the chain at the same point;
T the tension at any other point;
x any variable absciss;
y the corresponding ordinate;
s the corresponding arc;
h the weight of an inch, or a foot, &c. of the chain, x, y, s, &c. being taken in the same unit of measure.

This notation being established, the following are the principal properties of this curve; viz.

1. $\frac{\overset{\prime}{l}}{l} = \frac{cos.\ c}{sin.\ c}\ h.\ log.\ \frac{cos.\ c}{1 - sin.\ c}$.

2. $\frac{A\ sin.\ c}{h} = \frac{1}{2}\ l$, or $A = \frac{h\ l}{2\ sin.\ c}$.

3. $T = \sqrt{\{A^2 - 2\,A\,h\,s.\ sin.\ c + h^2 s^2\}}$.

Which, at the lowest point, becomes

$$T = A\ cos.\ c.$$

4. $\frac{1}{2}\ l = \frac{A\ sin.\ c}{h}$ and $\frac{1}{2}\ \overset{\prime}{l} = \frac{A\ cos.\ c}{h}\ h.\ log.\ \frac{cos.\ c}{1 - sin.\ c}$.

5. $x = \frac{A\ cos.\ c}{h}\ h.\ log.\ \left\{ \frac{A - h\,y \mp \sqrt{\{(A - h\,y)^2 - A^2 cos.^2 c\}}}{A\ (1 - sin.\ c)} \right.$

6. $\overset{\prime}{y} = \frac{A\ (1 - cos.\ c)}{h}$.

Where $\overset{\prime}{y}$ is the ordinate to the middle or lowest point of the curve:

7. $s = \frac{A\ sin.\ c}{h} \pm \sqrt{\frac{\{(A - h\,y)^2 - A^2\ cos.^2\ c\}}{h}}$.

These formulæ are not all necessary for the solution of our problem, but are given as embracing the principal properties of this curve.

Let us now suppose a bar of iron, which we must consider as flexible, to be fixed to two points of suspension, 1000 feet distant, the lowest point of the curve being $\frac{1}{20}$th of the whole distance, or 50 feet; and let it be required to find the length of the bar and its action on the points of suspension, the weight h of one foot of it being given. In the present question, assuming the specific gravity of iron 7788, and the diameter of the bar $\sqrt{}$ 18 inches, we find $h = 48$ lbs.:

also $\overset{\prime}{l} = 1000$ feet: whence, by formula 6,

$$\overset{\prime}{y} = \frac{A\ (1 - cos.\ c)}{h}, \text{ or}$$

$$A(1 - cos.\, c) = y' h = 48 \times 50 = 2400.$$

And formula 4 gives

$$\tfrac{1}{2} l' = \frac{A\, cos.\, c}{h}\, hyp.\, log.\, \frac{cos.\, c}{1 - sin\, c.} = 500.$$

$$\frac{2400\, cos.\, c}{48\,(1 - cos.\, c)}\, hyp.\, log.\, \frac{cos.\, c}{1 - sin.\, c} = 500.$$

Whence $\dfrac{cos.\, c}{10\,(1 - cos.\, c}\, hyp.\, log.\, \dfrac{cos.\, c}{1 - sin.\, c} = 1:$

which, by approximation, gives angle $c = 11° \; 15'$ nearly.

And hence, by formula 1, we find

$$l = 1008 \text{ feet} = \text{length of the catenary.}$$

Again, by formula 2,

$$A = \frac{h\, l}{2\, sin.\, c} = \frac{1008 \times 48}{\cdot 39018} = 124005\,\text{lbs. or,}$$

about 55 tons, the tension at the point of support.

Now we have seen, page 258, that the ultimate strength of iron, on a square inch, is 27 tons; and $18 \times \cdot 7854 = 14\cdot 1372$ inch, the area of section of the bar: which, for the sake of simplifying, and being on the favourable side with regard to our ultimate result, we shall call 14 inches; consequently,

$$14 \times 27 = 378 \text{ tons,}$$

for the ultimate strength of the bar, which from its own weight is only exposed to a tension of 55 tons: consequently, to find the load x, which the bar of iron is capable of bearing beyond its own weight, we have the equation

$$\frac{1008 \times 48 + x}{\cdot 39018} = 395 \times 2240,$$

or $x = 296847$ lbs. or 132 tons, which might be uniformly distributed over the bar besides its own weight, before a fracture would take place; and if we multiply this by the 16 bars, we shall have

$$132 \times 16 = 2112 \text{ tons,}$$

which might be uniformly distributed over the bridge besides its own weight, before the rupture would take place.

Now, Mr. Telford's estimate of the load the bridge will have to bear, at any time, is as follows:

		tons.	*lbs.*
Road way, 1008 feet in length	of fir timber	430	920
	of oak ditto	44	1440
Iron work connected with road way, also in suspending rods and railways		98	428
Weight allowed at one time upon the 1000 feet		100	0
Greatest weight suspended at one time		673	548

There will therefore, in the most extreme state of loading, remain a surplus strength of about 1438 tons.

With regard to the horizontal draw upon the top of the pier, this will, at each point, be in part counteracted by the two semi catenaries; and if the lowest point of these could be kept, as was first intended, exactly in the same horizontal line as the lowest point of the principal one, the action and reaction at each point would be exactly equal and contrary, and totally destroy each other's effects, so that the pier would theoretically stand unsupported; but if, according to the second plan, the semi catenaries have their origin lower than the centre of the principal one, the piers will have to support a horizontal force, which will have a tendency to make them fall inwards towards each other, and which will be so much the greater as the origin of the semi arcs descends below the horizontal level of the centre arc: it is, therefore, highly desirable that this difference should be as little as possible.

On Revetments.

33. Before we can expect to derive any useful practical information on the subject of revetments from theoretical investigation, certain courses of experiments become necessary, in order to establish the requisite data.

1. It is important to know the strength of cohesion of the materials commonly employed in the construction of these works; that is, of the different kinds of brick and stone, as also of the mortar or cement by which they are united into one mass.

2. We ought to know the natural slope of different kinds of soil, or the inclination their surfaces will assume when left unsupported.

3. It is proper that we should ascertain the degree of resistance or friction which takes place between the surface of the earth which remains undisturbed, and that part which slips down; because the natural pressure of the unsupported part will be diminished by a quantity equal to that resistance.

We have but few experiments from which to draw these necessary data; but, with regard to the natural slope of different soils, it has, by a rough measurement, been ascertained that, in the lightest kind of sand, the height of the bank is to the base of the slope as 5 : 4; and that, in soil of a closer and denser nature, the height is to the base in about the ratio of 7 : 5; and, probably, the natural slope of all varieties of soil, when newly converted into banks, &c. will be found to fall between the above limits. As to the resistances between the sliding and fixed surface, I am aware of no experiments that can be referred to, except those of Col. Pasley, and these are not exactly of this kind.

On the Cohesion of Brick, Stone, &c.

34. The only experiments that I know of relative to the cohesion of stone, are those of M. Gauthey, a German engineer; who has found, from the results of several trials, that a piece of stone, of what he denominates soft *givry*, 1 foot square and 1 foot long, required a weight of 5000 lbs. to

break it across, one end being fixed in a rock, and the weight hung on at the other; and that hard *givry* required 5600, under similar circumstances and dimensions, to produce the fracture.

In order to ascertain the cohesion of brick, 3 common bricks were procured, that had been exposed to the weather for 2 years at least; and 3 of the same kind, of recent make; and 3 of the *best stock*. These were supported between two props, 8 inches apart, and then loaded in the middle till they broke. The least thickness of the bricks was $2\frac{1}{2}$ inches, and the greatest 4 inches; and they were placed with their less dimension vertical. The following are the results of these experiments:—

	Common old Brick.	*Common new Brick.*	*Best Stock.*
	1......384 lbs.	1......411 lbs.	1......434 lbs.
	2......298	2......411	2......479
	3......347	3......387	3......420
Mean	3) 1029	3) 1209	3) 1333
	343 lbs.	403 lbs.	444 lbs.

As it is known, from the theory of the strength and stress of timber and materials generally, that the strengths of similar formed bodies, when exposed to transverse strains, are directly as the breadth and square of the depth, and inversely as the length; we shall find, by reducing the above means to 1 foot square and 1 foot long, when fixed at one end, that

Common old brick = 3939
Do. of recent make = 4631
Best stock ...... = 5115

To these we may add the results of Gauthey, viz.

Soft givry ...... = 5000
Hard do. ...... = 5600

With regard to the strength of mortar, we shall, for want of experiment, assume it equal to $\frac{1}{10}$th of the strength of the material itself.*

* It has been seen, art. 30, Appendix, that cement is stronger than the brick; but I do not know the strength of common mortar.

35. *On the Force necessary to overturn Walls and Columns.*

1. A column of soft givry (specific gravity 4000) is erected on a base 2 feet square, and its height is 20 feet. Required the force, acting perpendicular to its end, necessary to overturn it.

It is obvious here that the force necessary to produce the fracture will consist of two parts, viz. 1st, that which is necessary to produce an equilibrium with the weight of the wall, independent of the cohesion; and 2d, of a part sufficient to overcome the cohesion, independent of the equilibrium. The latter will vary with the area of the base of fracture and the point of application of the force; and the former with the weight of the column and the situation of its centre of gravity.

Generally, if W denote the weight of the wall, l the distance of the point of application of a direct force from the fulcrum about which the wall is to turn, and r the distance of the centre of gravity from the same, both in feet; then, by the property of the lever $F = \frac{W r}{l}$, the force necessary to produce an equilibrium.

And, from the theory of the strength of materials $\frac{F' l}{a d^2} = C$, *a constant quantity,* where a is the breadth, and d the depth of the section of fracture in feet; whence $F' = \frac{a d^2 C}{l}$, the force requisite to produce the fracture: therefore,

$$F + F' = \frac{W r}{l} + \frac{a d^2 C}{l}, \text{ the whole force required.}$$

In the present case, $W = 4000$ oz., or 250 lbs. $\times 2^2 \times 20 = 20000$ lbs., $r = 1$, $l = 20$, $a = 2$, $d = 2$, and $C = 500$; whence,

$$F + F' = \frac{20000}{20} = \frac{8 + 500}{20} = 1000 + 200 = 1200 \text{ lbs. the}$$

force sought.

Examples.

1. Suppose a column of the same dimensions as that above, of the best stock brick, (sp. gr. 3000,) is to be overturned by means of a rope fastened at its top, and brought to a distance of 40 feet from its base; required the power or weight necessary to upset the column?

2. A pyramid of the same base and altitude, and of the same materials, is to be broken or overturned by a direct force applied at its upper part; required that force, and in what part of the height the fracture will take place?

On the Pressure of Banks, and the Dimensions of Revetments.

36. Having established above (at least approximatively) certain data relative to the resistance and cohesion of walls and columns, it remains now to ascertain the pressure of earth against revetments, in order thence to determine their requisite dimensions, that an equilibrium may be established between those two forces.

For this purpose, let C B H E (*fig.* 12, *pl.* III.) denote a bank of earth, the natural slope of which is E B. Let the weight of the part C B E, one foot thick $=$ W, and make $\text{B E} = l$, $\text{C B} = h$, $\text{C E} = b$. From the theory of the inclined plane, as $l : h :: \text{W} : \frac{h}{l}\text{W} = \text{W}'$, the weight which, attached to the centre of gravity of the sliding solid, would preserve it in equilibrio on the plane E B, supposing no friction between the two surfaces. The weight W' will, therefore, under this supposition, denote the quantity, F I the direction, and I the effective point of application of the force of the bank against the wall A B C D. And now, to find the horizontal force at I: since the triangles K F I and B E C are similar, we have by the resolution of forces $l : b :: \text{W}' : \frac{b\,\text{W}'}{l} = \frac{b\,h\,\text{W}}{l^2}$,

for the horizontal effect at I: also, since K A, from the nature of the centre of gravity $= \frac{1}{3}$ of D A, or $\frac{1}{3}\ h$; and $\text{K I} = \frac{h\,x}{b}$, and $\text{A I} = \frac{1}{3}\,h - \frac{h\,x}{b}$, ($x$ being taken to denote the breadth of the wall at bottom,) the whole effect of the above pressure to turn the wall as a lever about a fulcrum at A, will be expressed by

$$\left(\tfrac{1}{3}\,h - \frac{h\,x}{b}\right)\frac{b\,h\,\text{W}}{l^2}, \text{ or } \left(\tfrac{1}{3}\,h - \frac{h\,x}{b}\right)\frac{b^2\,h^2\,s}{2\,l^2},$$

s denoting the specific gravity of the earth.

Now, to find the dimensions of the revetment requisite to keep this force in equilibrio, let h' denote the given height of the wall; S its specific gravity, or the weight of one cubic foot; x, as above, the thickness of the wall at the bottom; y the distance of the perpendicular, let fall from its centre of gravity upon its base, from the outward edge of the wall at bottom, viz. the point about which the wall turns; and a the area of its transverse vertical section; then, since we are only considering 1 foot in length, the same quantity, a, will also denote the solid content of the wall opposed to the bank; and, consequently, a S will be its weight.

Therefore, by the preceding proposition,

$$\text{F} = y\,a\,\text{S},$$

the resistance which the wall opposes in consequence of its weight, and

$$\acute{\text{F}} = \text{C}\,x^2,$$

the resistance from cohesion, C being a constant quantity, viz. $\frac{1}{10}$th of that belonging to the given material, as in the preceding proposition; whence

$$y\,a\,\text{S} + \text{C}\,x^2$$

will be the whole resistance opposed to the bank; and, consequently, in case of an equilibrium, or of an equality between

the force of pressure of the bank and the resistance of the wall, we shall have

$$y\,a\,S + C\,x^2 = \frac{b^2\,h^3\,s}{6\,l^2} - \frac{b^2\,h^3\,s\,x}{2\,b\,l^2};$$

a general formula, from which x, the breadth of the wall, in all cases may be determined.

If the wall be rectangular, then $y = \frac{1}{2}\,x$, and $a = h'\,x$, and the above becomes

$$\tfrac{1}{2}\,h'\,S\,x^2 + C\,x^2 = \frac{b^2\,h^3\,s}{6\,l^2} - \frac{b^2\,h^3\,s\,x}{2\,b\,l^2};$$

$$\text{or, } x^2 + \frac{b\,h^3\,s\,x}{S\,h'\,l^2 + 2\,C\,l^2} = \frac{b^2\,h^3\,s}{6\,C\,l^2 + 3\,h'\,l^2\,S}.$$

If the wall be triangular, then $y = \frac{2}{3}\,x$, and $a = \frac{1}{2}\,h'\,x$, and the above becomes

$$\tfrac{1}{3}\,h'\,S\,x^2 + C\,x^2 = \frac{b^2\,h^3\,s}{6\,l^2} - \frac{b^2\,h^3\,s\,x}{2\,b\,l^2};$$

$$\text{or, } x^2 + \frac{b\,h^3\,s\,x}{\frac{2}{3}\,h'\,l^2\,S + 2\,C\,l^2} = \frac{b^2\,h^3\,s}{2\,h'\,S\,l^2 + 6\,C\,l^2}.$$

Ex. 1.—As an example, let the natural slope of a given soil, when unsupported, be 45°, and its specific gravity 2000, or the weight of a cubic foot, 125 lbs.; and let it be required to determine the breadth of a rectangular wall of soft givry necessary to support it: the wall and bank being both 12 feet high; and the specific gravity of the wall 2500, or 156 lbs. to the cubic foot.

Here $h' = 12$, $h = 12$, $b = 12$, $l = 12\sqrt{2}$, $S = 156$, $s = 125$, and $C = 500$.

Whence,

$$x^2 + 3{\cdot}794\,x = 15{\cdot}176;$$

or, $x = -\,1{\cdot}897 \pm \sqrt{(1{\cdot}897^2 + 15{\cdot}176)} = 2{\cdot}435$ feet.

Ex. 2.—Let all the data remain the same, to find the breadth at bottom of a triangular wall, that will keep the same bank in equilibrio.

Here, putting our second formula into numbers, we have

$$x^2 + 5{\cdot}148\,x = 20{\cdot}594;$$

or, $x = -\,2{\cdot}574\,x + \sqrt{(2{\cdot}574^2 + 20{\cdot}594)} = 2{\cdot}643$ feet.

This is but little different from the former, as ought

obviously to be the case, because a great part of the resistance is due to the cohesion of the bottom section, that arising from the weight being comparatively small: It is singular, therefore, that the former datum has never (I believe) been introduced into the solution of the problem. Prony, who has attempted an elaborate solution of this proposition, has no reference to the wall's cohesion. It will be observed, also, that in the above investigation we have not considered the friction of the two surfaces: this is, of course, very considerable, and will reduce the thickness of the wall to a quantity less than the above. Experiments are, therefore, necessary to establish this point: in the mean time it may be observed, that, as it is always desirable that the resistance of the wall should be more than equal to the pressure it has to sustain, it will be safer to omit it entirely than to introduce it without very correct data, drawn from the results of experiments carried on upon a large scale.

Ex. 3.—Supposing the wall to be built of the best stock brick, which weighs 100 lb. to the cubic foot, and that a cubic foot of the earth weighs 96 lbs.; also, that the bank is 12 feet high, and the natural slope of the soil is 30°: what must be the thickness of the rectangular wall that will just prevent the bank from slipping?

Ex. 4.—With the same data, required the thickness of the wall at bottom, supposing it in the form of a triangular wedge, as in the second example above?

Ex. 5.—To find the thickness of an upright rectangular wall necessary to support a body of water, the depth being 10 feet, and the wall 12 feet high, the specific gravity of water being 1000, and the best stock brick 2000.

Ex. 6.—Required the thickness of the wall at bottom, supposing the data the same as in the preceding example, but the wall to be in the form of a triangle, as in examples 2 and 4?

Note.—The pressure in the last two examples is to be estimated on the principles of the pressure of fluids.

Remark.

The above can only be considered as a very imperfect sketch of the theory of Revetments, at least as relates to its practical application, for want of the proper experimental data: being merely given, in connexion with our general theory of the strength of materials, for the sake of introducing considerations relative to the cohesion of walls, &c., which have been commonly omitted: and the consequence has been, that, according to all theories, (and there have been several,) the computed thickness of the wall has very far exceeded what was ever considered to be practically necessary.

To render the theory complete, with respect to its practical application, it is necessary to institute a course of experiments upon a large scale; first, upon the strength of common cement and mortar; and, secondly, upon the force with which different soils tend to slide down, when erected into the form of banks. A well-conducted set of experiments of this kind would blend into one what many writers have divided into several distinct data. Thus some authors have considered first, what they call the natural slope of different soils, by which they mean the slope that the surface will assume when thrown loosely in a heap; very different, as they suppose, from the slope that a bank will assume that has been supported, but of which that support has been removed or overthrown. This, therefore, leads to the consideration of the friction and cohesion of soils, and what is denominated the slope of maximum thrust: but however well this may answer the purpose of making a display of analytical transformations, I cannot think it is at all calculated to obtain any useful practical results. I should conceive that a set of experiments, made upon the absolute thrust of different soils, which would include or blend all these data in one general result, would be much more useful, as furnishing less causes of error, and rendering the dependent computations much more simple and intelligible to those who are commonly interested in such deductions.

We may further observe, that the method of resolving force of the bank at the point I, instead of the point which former is obviously the effective point as regards lever by which the wall turns, shews, that while the c tinuation of the slope falls within the base of the wall, soil which forms it will add to the stability of the revetme which is conformable to the experiments of Colonel Pas See Vol. III. of that author's " Course of Military Instr tion."

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